63-4-2

408 655

REPORT:

MODIFIED LINEAR PROGRAMMING

Submitted to:
Department of the Navy
Bureau of Supplies and Accounts
Research and Development Division
Washington, D. C.

By:
Alderson Associates, Inc.
Philadelphia, Pennsylvania
June 1960

TABLE OF CONTENTS

SUMMARY: FINDINGS AND RECOMMENDATIONS

1. Under present arrangements the number of large scale transportation routing problems arising out of SPCC redistribution actions is relatively small. A sample indicates that less than five percent of these problems are of sufficient size and complexity to merit sophisticated computational treatment. For these purposes a problem was considered to be sizable if it involved no less than seven activities, no less than three consignees and no less than three consignors.

2. However, it is possible to effect worthwhile savings in transportation outlay on those larger redistribution problems. Even approximative techniques which involve no more complex calculations than the present rules can save about two percent on transportation mileage in the average redistribution problem. This is a relatively small figure but considering the large annual outlay on redistribution transportation cost the absolute dollar saving is likely to be considerable.

3. Most of our calculations were based on straightforward mileage tables rather than transportation cost tables or the proximity tables currently in use by SPCC.

a. Transportation cost figures were not employed because they are not available in a form which is directly usable. It would be impracticable to attempt to construct and compute with a separate cost table for each of the many thousands of items handled, and so it would be necessary to use a classification of items into a small number of classes of similar weight and bulk with transportation costs being given for each such class. These data are not available at present.

b. The proximity tables were not employed because, by design, they are neither a pure index of geographic proximity, nor an indicator of actual transportation costs. Rather they represent an embodiment of the traditional practices and rules of thumb which have been developed at SPCC.

4. As explained in the body of this report, transportation problems which "degenerate" permit reductions in the number of shipments required by a redistribution, and therefore allow savings in fixed charges. However, at SPCC the number of problems which happen to be degenerate without any adjustment in excess and requirement figures seems to be small. In a sample of problems specially selected as likely to involve degeneracy, less than one in seven turned out to be usefully degenerate and none of them permitted any substantial reduction in number of shipments. Moreover, because the computation is usually difficult and time consuming, searching for degeneracy is not a practical method for dealing with fixed charges.

5. A degeneracy forcing computing procedure was therefore developed to deal with the fixed charges problem. The purpose of the procedure is to introduce artificial degeneracy into the problems in order to effect an overall reduction in the number of line items redistributed (the number of individual shipments made). This should reduce overall fixed charges—the administrative, clerical and delay costs which arise out of an increase in the number of shipments but which are unaffected by the size of any individual shipment. The computing procedure is flexible and can readily change the weight it gives to fixed charges relative to transportation costs in its calculations. Clearly, the extent to which fixed charges should be permitted to affect the final solution depends on the magnitude of these costs. As yet we have not been supplied with definitive figures on their size. In an extremely conservative experimental computation the computational method reduced the number of shipments by 9 percent for an average problem and at the same time changed transportation mileage negligibly from its optimal level!

6. We believe that there is a possibility that far greater savings in both transportation costs and fixed charges can be obtained in the long run by a variety of other more fundamental approaches, including the following possibilities each of which merits careful analytical investigation:

a. A well defined policy which distinguishes between an optimal minimum inventory level below which an activity is declared to be in a requirement position and an optimal maximum inventory level beyond which the activity is considered to have an excess. Only in this way can the system be prevented from shipping goods in response to the slightest demand change and thus keeping a substantial proportion of the Navy's inventory "travelling about the country in boxcars." (Substantial theoretical work on this sort of inventory policy in a fixed charges situation has been done at Princeton by D. Orr in terms of the theory of random walks.)

b. In the long run, substantial savings can be expected from a dynamic analysis which takes account of the feedback properties of the system--the interrelation of demand patterns, redistribution shipments and present and future inventory levels. Failure to take these complex relationships into account will result in failure to adjust to foreseeable future requirements, and perhaps even more important, it is likely to produce artificially induced and unnecessary excesses and requirements in the future.

7. We therefore recommend:

a. That, at least on a trial basis, SPCC install the forced degeneracy approximative transportation algorithm (as based on the SMALC method) as part of its redistribution calculation as soon as possible. During the trial period its results should be watched carefully by the clerical staff and by Alderson Associates, but interference with its operation should be kept at a minimum.

b. The cost figures to be used as a basis for the calculation should be a straightforward distance ordering of all activity pairs, modified by any appropriate considerations which are now reflected in the proximity table.

c. Further information-gathering efforts under the sponsorship of BUSANDA should be designed explicitly in terms of the several mathematical models and algorithms which have been designed under its research program. In this way it is likely that much more explicit information can be obtained on the values of the parameters which must be known for most effective use of the models.

d. Research on the transportation and redistribution problems should be continued. However it should begin at a more fundamental analytical level than the present study and it should proceed along the lines indicated under item 6 above.

I. Assignment

In July of 1959 Alderson Associates, Inc. was autherised to proceed on a research project for the Bureau of Supplies and Accounts of the United States Navy. The project was given the title "Modified Linear Programming." Specifications for the project as prepared by the Bureau read in part as follows:

"This project task is directed toward development of more efficient rules for the distribution and redistribution of material in the Navy supply system, through modification of the techniques of linear programming to represent adequately and minimize the total costs of alternative allocation and redistribution decision patterns for Navy material. In particular, the project seeks to discover feasible and workable approximating rules for distribution decisions where a fixed cost of shipment is postulated for each 'channel' in addition to the customary variable cost, linear with respect to quantity. The technique of integer programming and several other short-cut methods appear to offer promising approaches....

"The problem is being examined in the context of Ships Parts Control Center's present supply decision and data reporting structure: 30-odd stocking points reporting individual item stock transactions daily, with weekly summarization and review on IBM 705 equipment; computation of gross requirements and net excess-or-required position for each activity and system buy-or-no-buy position; fixed costs for a shipment approximately the same for all activities, with linear or piecewise linear variable costs based on distance and transportation rate data....

"It is expected that the models will eventually incorporate rough measures of uncertainty and changes over time in demand patterns. Emphasis in this project is placed upon usable rules and approximations rather than precisely minimal decision solutions.[1]"

In retrospect, this description of the project turns out to be more farsighted than one has a right to expect of any specifications written for a pioneering research project. Though Alderson Associates was prepared to modify its approach to the fixed charges transportation problem as the need arose, the procedures employed turned out to match BUSANDA's description almost exactly and step by step.

Several of the specifications for the study which are stated or implied by this description require emphasis.

1. The project involves a problem of transportation routing;

2. A distinguishing feature of the study is that it is to take into account the fixed charges which arise out of redistribution actions, where these fixed charges are "approximately the same for all activities";

3. Approximative methods of solution are to be considered as a means for saving costs and economizing on the use of crowded computer facilities which are likely to preclude the use of a full scale programming computation in light of the vast numbers of such problems handled by the Navy supply system.

4. In the current stage of the investigation, requirements and excess figures for each relevant activity (i.e. the amounts to redistributed) are to be

---

[1] Bureau of Supplies and Accounts, Research and Development, Technical Progress Report, Washington, D. C. (Report dated August 26, 1959.)

are to be taken as given and the method of their determination is not to be examined with a view to their possible modification.

5. However, the study is to be aimed toward an eventual incorporation of demand patterns, uncertainty (and, presumably, the resulting dynamic structure) which can only mean that the excess and requirement figures will then, along with the transportation routes, become central variables in the analysis.

II. The Need for Approximative Calculations

Our study indicates that approximative techniques are indeed appropriate for the handling of the problem as had originally been envisioned. However, the reasons turn out to be somewhat different from those anticipated.

The number of redistribution calculations in the Navy supply system is tremendous. The Ships Parts Control Center alone handles over 150,000 items, of which some 20-30,000 require review of their stock position about every three weeks. In the average review, about 7,000 line items will show a redistribution action. This profusion of redistribution calculations suggests that a full scale simplex method or network linear programming calculation is likely to be impractical. Even with a relatively efficient and speedy program the amount of time required may rapidly add up. A recent study at one Navy supply installation suggested that as much as forty hours of computer time per review period might be required by an ordinary transportation calculation which took into account no complications such as fixed charges or uncertainty. Certainly such a computation time requirement would be prohibitive considering the cost of computer time and the many other problems competing for Navy computer facilities.

Nevertheless, at one stage in the course of the study it seemed possible that practicable non-approximative methods of solution might be obtained. The Ford-Fulkerson network method of dealing with the transportation problem[1/] has been able to achieve remarkable computational speeds and it was thought that an efficient program might obviate the need for approximation.

1/See L. R. Ford and D. R. Fulkerson, "Solving the Transporation Problem," Management Science, Vol. 3, October, 1956 and (same authors) "A Primal Dual Algorithm for the Capacitated Hitchcock Problem," Naval Research Logistics Quarterly, Vol. 4, March, 1957.

It is still conceivable that for some of the Navy supply installations this may prove to be the case. However, experience at SPCC suggests that it is unlikely. There are two reasons why a full programming calculation is apt to be impractical.

1. The computing equipment employed by the Navy supply system consists largely of accounting and business-oriented machines rather than computers designed primarily for scientific research calculations. For example, the IBM 705 Mark III used at Mechanicsburg is a business machine version of the more flexible 704. Unfortunately for present purposes the specialized accounting-oriented computers are not well suited for easy and efficient programming of problems of the sort under consideration. This is not necessarily a criticism of the computing machines which the Navy has chosen to install. Rather, it suggests that the other needs of the supply system have led to the installation of specialized equipment which is not well adapted to fast non-approximative methods of linear or nonlinear programming computation.

2. A second source of the continued emphasis on approximation is the heavy demand on the available computer memory space. Of course, the computers have many tasks other than the determination of transportation routing. Moreover, many of these other computations constitute integral parts of the periodic redistribution review. The result is that, at least at SPCC, only a small number of memory locations are available for the redistribution routing decision and this effectively precludes even moderately complex calculations. For practical purposes this rules out any procedure requiring substantially more instructions and more elaborate data processing than the current proximity table approach.

III. Fixed Charges and their Consequences for Efficiency

In addition to the problem of finding a good approximation technique, a second, and more difficult problem is that which arises out of the presence of fixed charges--charges which are incurred whenever a redistribution action is taken but which do not vary with the amount of material involved in a particular shipment. A prime example of this sort of cost arises from the preparation of some of the papers which are required in the course of such an action. Thus, if a shipment is eliminated altogether, the cost of invoice preparation is avoided. But once a shipment is undertaken, the cost of making out the invoice is not substantially affected by a decision to send 200 cases rather than 10 cases of the item.

Information on the magnitude of these fixed charges is still rather limited so it is difficult to assess their importance to the Navy supply system. However, it is clear that they can arise in at least two different ways:

1. They arise out of the clerical and administrative work associated with any shipment. However, it has been pointed out that in the short run the elimination of this sort of fixed cost may result in relatively little cash saving to the Navy. If one hour of clerk time is saved per day per activity it is very unlikely that any personnel reduction will occur. But, of course, in the long run a sufficient accumulation of such savings can lead to a decrease in clerical outlays by reducing the number of clerks who must be hired as replacements or additions to existing staff.

2. A second type of fixed cost has been pointed out by the Dunlap Study. An increase in the number of shipments can result in a slowing down of commodity movements. If paperwork is a bottleneck, an additional redistribution action can reduce the speed with which others can be processed.

This reduction in speed may then be dependent on the number of such actions rather than on the magnitude of the shipments involved. Consumer waiting time may, therefore, very well be a fixed charge. The information available on this type of fixed charge is as yet very sketchy, and it certainly merits further investigation.1/

If fixed charges are left out of account of a transporation analysis, costly miscalculations are likely to arise for the following reasons:

1. Use may be made of shipping routes which incur fixed costs sufficiently large to offset whatever other advantages these routes may offer.

2. Too many separate shipments may be made despite the fact that each additional shipment adds to the number of fixed cost charges. That is, the presence of fixed charges tends to call for a smaller number of (larger) shipments than would otherwise be the case.

3. Small and unimportant shipments may be made because their variable cost alone is likely to be insignificant, despite the fact that they may not be worth the fixed cost which they incur.

1/The distinction between the two types of fixed costs may have one important consequence. Paperwork procedures are fairly similar throughout the activities of SPCC and any other SDCP or segment of the supply system. As a result these fixed charges are likely to be approximately similar at all activities, as we are currently assuming. However, the crowding of clerical facilities is likely to differ from activity to activity (although this is apt to be a non-optimal situation; if these differences are persistent, provision to change them should, perhaps, be made). If crowding differs, the delay fixed costs may well vary from activity to activity.

The standard transportation calculation of linear programming takes no account of these fixed costs and can, therefore, result in serious errors of the sort just listed. That is why our major research objective has been the development of a modification of the transportation calculation which takes these fixed costs into account.

IV. The Fixed Cost Transportation Problem: Mathematical Model

Let us turn now to a description of the basic mathematical model which was employed in this study. It will be noted that it differs from the standard transportation model in two respects. First, it explicitly includes both procurement and disposal processes as well as ordinary shipments over the available routes. However, this makes very little difference to the mathematical structure since procurement simply involves the addition of one (or several) ficticious "activities" which are always in a sufficient excess position to constitute a possible source for activities with deficits. Disposal can be handled similarly.

A second special characteristic of the model lies in the structure of its coefficients which reflect the nature of the fixed charge by the following standard device. The costs are divided into two parts, K and CX, where K is the fixed charge, C is the (variable) cost per unit shipped, and X is the amount shipped.

But K is not a constant. Rather, it can take either of two values, If $X = 0$ (nothing shipped over the particular route) then K also becomes zero (the fixed charge disappears.) However if any amount, however small is shipped ($X > 0$) then K takes its fixed value, for which we use the symbol K^*.

The model employs the following notation.

We let $X_{i,j}$ be the quantity of commodity X redistributed from activity i to activity j ($i = 1,2, \ldots, N$, $j = 1,2, \ldots, N$)

where "activity" $N-1$ is a fictitious procurement activity (a supplier of goods) and N is a fictitious disposal activity (a receiver of goods).

also let

E_j be the excess stock of X (positive or negative) at activity j,

where this is either a given constant or a random variable with known distribution.

$K_{i,j} + C_{i,j} X_{i,j}$ be the total cost of redistributing $X_{i,j}$

where:

$C_{i,j}$ is presumably a fixed constant,

$K_{i,j} = K_{i,j}^* > 0$ (a constant) if $X_{i,j} > 0$,

and

$K_{i,j} = 0$ otherwise.

Using this notation, the problem may be formulated as follows:

The objective is to minimize the redistribution cost of commodity X, i.e., to

Minimize $\quad \sum_{i,j}(K_{i,j} + C_{i,j} X_{i,j})$

Subject to

$$\sum_j X_{i,j} \leq E_i^* \qquad i = 1,2, \ldots, N-2$$

(No activity ships out more than its excess stock)

and

$$\sum_i X_{i,j} \geq E_j^{**} \qquad j = 1, 2, \ldots, N-2$$

(No activity receives less than its requirement)[1/]

and where $\quad$ all $X_{i,N-1} = X_{N,j} = 0$

(No shipments into the "procurement activity" or out of the "disposal activity")

and all $X_{i,j} \geq 0$

1/In the ordinary transportation problem the inequalities are replaced by equations which in fact was done throughout most of the computations. The inequalities appear here to permit choice between procurement and redistribution or redistribution and disposal (e.g., not all of an activity's surplus will be shipped if it is cheaper to supply a requir ment of some other activity by purchase). Actually, these inequalities cause no serious computational problems.

Here we define

$$E_i^* = \begin{cases} E_i \text{ if } E_i > 0 \\ 0 \text{ otherwise} \end{cases}$$

$$E_j^{**} = \begin{cases} E_j \text{ if } E_j < 0 \\ 0 \text{ otherwise} \end{cases}$$

i.e. activity i is (by definition) a shipping activity only if it has a positive excess ($E_i > 0$) and activity j is a receiving activity only if it has a negative excess, i.e., a requirement ($E_j < 0$).

There remains only one detail to be specified in the description of this model. The constants which arise in procurement and disposal cost structures are somewhat different from those of the other processes. As compared to an act of redistribution, the procurement of a unit of commodity X involves a unit addition to the overall inventory held by the system. Therefore, this purchase will result in an addition to future carrying cost of the system. These carrying costs must be discounted (at the appropriate discount rate, D) and added to obtain their present value. By the usual discount formula, then, if S is the unit carrying cost of X per unit of time this become $S + DS + D^2S + \ldots = S/1-D$. We may then specify the costs of procurement and (similarly) those of disposal more closely as follows:

$$[illegible]_j([illegible]_{-1,j} \; [illegible]_{-1,j} \; [illegible]_{-1,j}) = [illegible]_j[[illegible]_{-1,j} + ([illegible]_j + \frac{[illegible]}{1-[illegible]}) [illegible]_{-1,j}]$$

where P_j is the unit cost of procurement (including transportation, etc.) for activity j,
S, is the unit carrying cost (per unit of time) and
D is the discount rate.

Similarly, the cost of disposal is given by

$$[illegible]_i([illegible]_{,i} + [illegible] \; [illegible]_{1,i}) = [illegible]_i[[illegible]_{,i}(V_i + \frac{[illegible]}{1-D}) [illegible]_{1,i}]$$

where V_i is the unit return from disposal.

This, then, is the basic model. It will play an essential role in the derivation of a crucial theorem later in this report.

V. Computational Problems Resulting from Fixed Costs

The fixed cost coefficients $K_{i,j}$ in the preceding model may appear innocuous enough. However they can lead to most serious computational and analytical difficulties. They transform the transportation model from an ordinary linear programming problem into a nonlinear problem, and, indeed, a nonlinear problem of a particularly intractable variety. To explain the nature of the difficulty it is necessary first to digress into a discussion of nonlinear programming.

In a nonlinear program the algebraic expressions which occur in either the objective function (the cost relationship) or in the constraints or both will involve nonlinear terms. Thus, rather than representing the cost relationship by a simple linear equation such as $5X + 3Y + 7Z$ we use the more general functional notation total cost, c, = $f(X,Y,Z)$ which states simply that cost is dependent in some unspecified way on the quantities involved in the three shipments, X, Y and Z. Similar notation is used for the constraints, so that the general nonlinear programming problem may be written in the usual three parts:

i. Objective function:

$$\text{Minimize } c = f(X,Y,Z,\ldots)$$

subject to

ii. constraints:

$$g_1(X,Y,Z,\ldots) \leq c_1$$
$$g_2(X,Y,Z,\ldots) \leq c_2$$
$$\cdots\cdots\cdots\cdots$$
$$g_m(X,Y,Z,\ldots) \leq c_m$$

and

iii. the nonnegativity requirements:

$$X \geq 0,\ Y \geq 0,\ Z \geq 0, \ldots$$

For fairly obvious reasons, the graph of a nonlinear objective function cannot be a plane as in the linear case. Instead, cost functions may be hills or valleys or of totally irregular shape.

Several such cost relationships (total cost as a function of X_a and X_b--the amounts shipped along two routes, A and B) are illustrated in the Figures 1 and 2. Figure 1 represents the "best behaved" type of cost function.

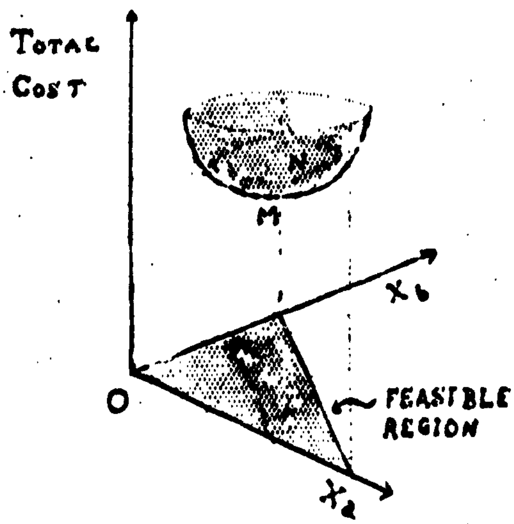


Figure 1

For reasons which are discussed presently, such a function makes life easier for the programmer than it is in the presence of other types of nonlinear cost functions. This "best behaved" type can be described as a diminishing returns case--the curvature of the surface (its U shaped cross sections) indicates that increases in shipments yield diminishing marginal returns. Indeed, increases in shipments beyond the cost minimizing point M, say to N, must yield diminishing <u>total</u> returns, i.e. such an increase in amounts sent must obviously increase total costs.

By contrast, Figure 2 in which the lowest points of the diagram occur over the axes of the diagram rather than at its center as in Figure 1, depicts an important case, at the other extreme, that of increasing returns.

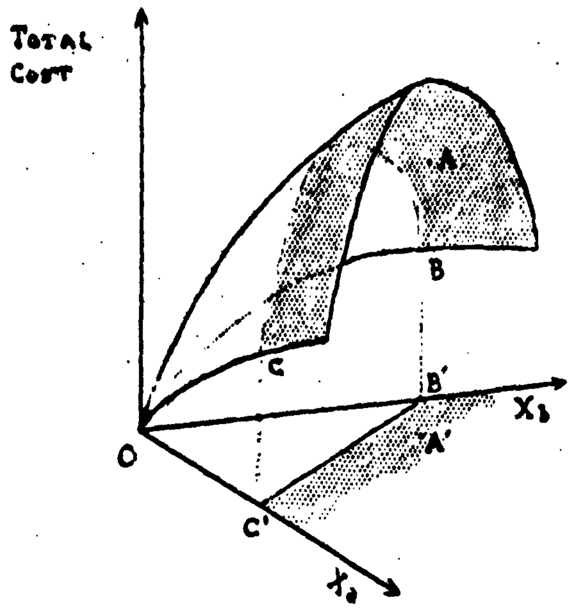


Figure 2

Figure 2 represents a case of increasing returns to specialization because it shows that lower costs can be achieved by exclusive use of route A (a point on the OX_a axis) or by exclusive use of route B than by use of a combination of these routes (a point toward the center of the X_aX_b plane).

The distinction between diminishing and increasing returns is extremely important for programming. In a diminishing returns case as that in Figure 1, if we find a point such as M from which any small move increases costs, we can be sure that this point is a global optimum, i.e., no move of any magnitude will bring in costs lower than those at M.

Because of the curvature of the cost surface, it moves further and further above its minimum point M. Thus if a small move away from M, say to N, increases costs, we can be sure that any further move in that direction, say to Q, will only increase costs still further.

However, this result does not hold for the increasing returns case depicted in Figure 2. There a move from point B over to A does indeed increase costs. But if we are patient and nevertheless continue to move along the surface it will begin to curl back downward again and eventually we may even reach a point, C, which yields costs far lower even than those at point B. A point like B whose costs are lower than those of any other feasible point in its vicinity is called a local optimum, whereas the point C which really yields minimum costs, is a global optimum.

What is the significance of this result? It states, in effect, that any computing procedure which identifies an optimum point by seeing whether a small move in any direction increases costs cannot be trusted in problems in which the objective function exhibits increasing returns. An example of such a procedure is the simple requirement of the differential calculus that the second derivative of the function whose value is to be maximized be negative at the maximum point. For this condition merely states that any move to a point very near the maximum point results in a reduction in the value of the objective function, and so it is a satisfactory condition only where increasing returns do not occur.

Directly related to the foregoing problem is the difficulty of finding a workable iterative procedure for getting to the global optimum point in an increasing returns case--for here the rule "proceed by successive steps each of which reduces costs" is not trustworthy. Thus, in Figure 1 no such problem arises; we can confidently proceed by moving downward in any direction because all downhill paths end up at the minimum point. Hence any trial and error (iterative) procedure which keeps trying successive output levels which are less costly than those in the previous attempts will (if it does not move up too slowly) eventually get us to the minimum cost shipping route combination.

But if in a cost minimization problem the graph of the objective function contains a hill (it exhibits increasing returns), going in a downhill direction is not guaranteed to get us to the lowest point. If we start downhill from point A in Figure 2 we may end up at point B instead of point C, the global optimum in the shaded feasible region.

To summarize then, many, indeed, almost all of the standard optimality calculation procedures are applicable only to problems involving constant or diminishing returns.

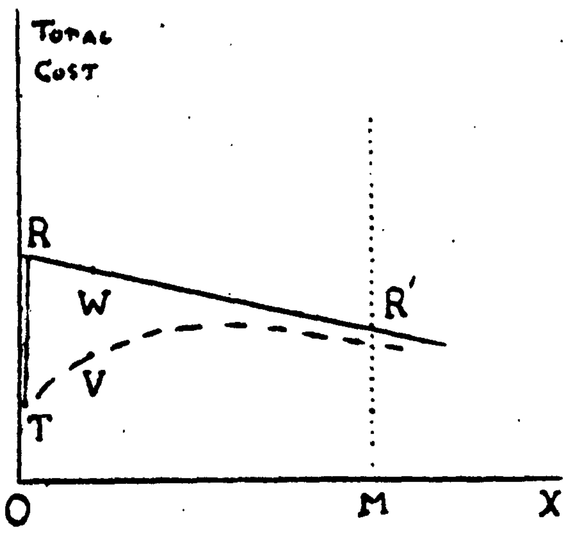


Figure 3a

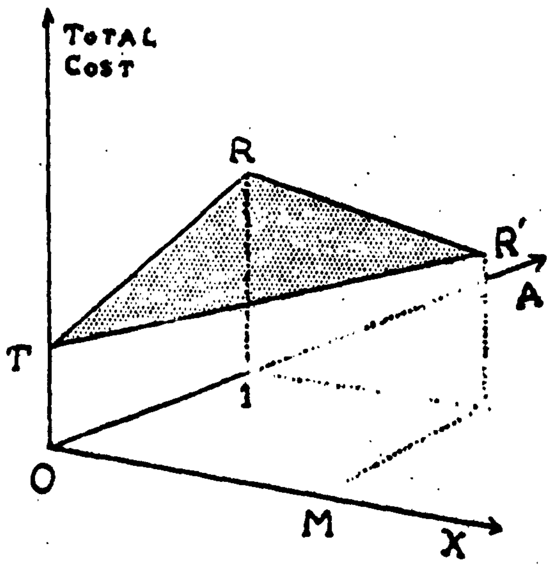


Figure 3b

Let us now return to the problem of fixed charges to see the relevance of the preceding discussion to the difficulties caused by increasing returns in nonlinear programming. Figure 3a represents part of the cost function of a multi-activity enterprise showing how its costs will vary when the amount shipped from one of its activities (b) varies, the amount shipped out of all other activities being given. This relationship is cost curve TRR'. As the diagram shows, if this activity does any shipping at all, the larger the amount which it sends out the smaller will be total costs (RR' slopes downhill toward the right). But in the case shown, if the shipping route is eliminated altogether, the fixed costs which it escapes are so large that costs will suddenly fall from R to T. In fact (assuming that there is some upper limit, OM, to the demand for this product) even if the activity sends out every bit that can be used by the recipient, the economies which are achieved will not suffice to make up for the fixed cost. This is because point R', whose height represents total cost at the maximum saleable shipment, lies above T, where OT represents the total cost when the activity stops shipping along this route altogether.

We see, then, that point R' is a local minimum but T is the global minimum. Indeed Figure 3a clearly represents an increasing returns objective function--it is a two dimensional relative of the cost function illustrated in Figure 2. But in the fixed charges case (Figure 3b) it is to be noted that <u>any</u> computation which tells us to go downhill along the cost curve will move us in the wrong direction. Even at a point like W which is very close to R there is not the slightest hint in the slope of the curve that costs can be reduced by reducing output. This is a particularly nasty feature of the fixed charges problem. An ordinary increasing returns cost curve, such as curved line TVR, will at least indicate the direction of the global minimum point when we get close enough to it--at point V, going downhill takes us toward global optimum T, even if starting further to the right the "go downhill" rule would take us in the wrong direction.

It is, of course, only because we are dealing with a multi-activity operation that our problem is really difficult. As a result, even our graph is likely not to give us the right answer. Perhaps it is best not to stop shipping out of activity B after all. Instead it might be better to eliminate deliveries from some other activity (C) and save the fixed charges at C. Meanwhile replace C's former deliveries by shipments from B for this increases the maximum demand for activity B and so permits us to move lower along the cost curve to the right of point R. With a large number of branches the problem of examining the possibilities case by case, to decide how many and which to choose, leads to an enormous problem of permutations and combinations which rapidly grows astronomical. A more systematic computation procedure is required.

The role of integer programming in such a problem is easily represented schematically. For this purpose it is necessary to introduce an artificial variable, A. In the three dimensional Figure 3b point T is placed where $A = 0$, while line RR' is moved to where $A = 1$. The three points T, R and R' are then connected by the plane TRR' which can now serve as the feasible portion of an artificial <u>linear</u> programming objective function. But if we include the constraints $A \geq 0$ and $A \leq 1$ in the problem and require that A take only integer values, it is clear that we can only have either $A = 0$ or $A = 1$. We can end up only at point T or on line segment RR', i.e., we must remain somewhere on the original cost curve TRR' of Figure 3a. Thus by use of integer programming we have been able to substitute for our original fixed charges problem another ordinary linear programming problem which gives the same answers.

In principle, this translation can be made for all of the Navy's activities at once and so the entire problem can be transformed into one large linear integer programming problem and thus be solved. Unfortunately, in practice this has not, so far, proved practical for even moderately large scale problems where the number of artificial variables which must be added can make the computation prohibitively time consuming and expensive. Recent modifications in Gomory's algorithm appear to be promising but, for the moment at least, some alternative methods must be explored.

VI. Equality of Fixed Charges and Degeneracy

The fixed charge computation is, however, considerably simplified if it can reasonably be assumed that all fixed charges are approximately the same. There is a theorem which states that in this case, unless the problem is degenerate the ordinary linear programming solution will in fact be optimal, i.e., the solution will be the same whether or not fixed charges are present! 1/

1/Proof: In the absence of degeneracy the solution to our fixed charges programming problem must have at least as many non-zero elements as there are constraints (2n in the case of our model). For (ignoring the possibility of inequality) let us denote our constraints by $a_{1,1}X_{1,1} + \dots + a_{nn}X_{nn} = b$ where the a's and b are column vectors of constants. If there exists a solution where only $M < 2N$ of the X's are non-zero it follows that some subset of M of these columns are linearly dependent, i.e., the problem must be degenerate.

If the fixed charge constants $K_{ij}{}^*$ are all equal to the same number K^*, the number of these fixed charges must be equal to the number of non-zero shipments (the number of positive X's), i.e., the fixed charges must add up to at least $2NK^*$.

Now, the objective function of our problem is $\Sigma (K + C_{i,j} X_{i,j})$ while the objective function of the ordinary linear programming problem (in the absence of fixed charges) is $\Sigma C_{i,j} X_{i,j}$. It is well known that the linear programming problem has an optimal solution X^* which contains exactly 2N non-zero elements. Hence, for these values of the X's we will have $\Sigma K = 2NK^*$, i.e., both ΣK and $\Sigma C_{i,j} X_{i,j}$ will be at their minima. Thus the linear programming problem solution X^* must also be the solution to the minimum fixed charges transportation problem.

One minor qualification must be added to the argument--in the normal transportation problem the number of routes employed will be one less than the number of constraints. That is because the problem is so set up that the total amount shipped equals the total amount demanded--so that if all but one customer's demands are satisfied the remaining customer's requirements must automatically be satisfied and his demand constraint becomes redundant. Cf. Warren M. Hirsch and George Dantzig, The Fixed Charge Problem; Rand Corporation Paper P-648, December, 1954.

The intuitive ground for this result is easily grasped. For reasons which will soon be explained, in the absence of degeneracy the minimum number of shipping routes which will get rid of all excesses and supply all requirements is fixed. This minimum number of routes will be employed in any basic optimal solution of the ordinary linear programming transportation problem. In this case fixed charges cannot be avoided by a reduction in the number of routes employed and, since all routes involve the same fixed charges, nothing is to be gained in this respect by choosing one route as against another. It follows that in such a case nothing can be done about reducing fixed charges--the best solution consists in just keeping variable costs down as low as possible--and the ordinary transportation algorithm will indicate how this can be done.

Clearly then, if fixed charges are approximately equal for all shipping routes, the only hope for cost reduction lies in degeneracy. Only degeneracy will permit fixed charge savings by making it possible for a reduction in the number of routes employed.

The last thing to be discussed in this expository section of the report is the relevance of degeneracy to the current problem.

In a transportation problem degeneracy is defined to mean that some subset of the activity requirements adds up to the sum of some subset of activity excesses. For example, if activities A and B need, respectively 27 and 9 cases of X while some other activity C has a surplus of exactly

36 cases, then provided there are other activities in surplus or deficit positions, the problem is degenerate because 36 = 27 + 9.[1]

---

[1] Actually this is a special case of the general linear programming definition of degeneracy, as a case in which a number (smaller than the number of constraints) of columns of coefficients in the constraint set are linearly dependent. To illustrate this, consider the two-shipper-two-destination transportation problem which has the following constraints:

$$
\begin{array}{llllll}
X_{11} & +X_{12} & & & = E_1 \\
 & & X_{21} & + X_{22} & = E_2 \\
X_{11} & & + X_{21} & & = R_1 \\
 & X_{12} & & + X_{22} & = R_2,
\end{array}
$$

where E_1 and E_2 are the excesses of surplus activities 1 and 2 and R_1 and R_2 are the requirements of the two deficit activities. The matrix of coefficients is

$$
\begin{bmatrix}
1 & 1 & 0 & 0 & E_1 \\
0 & 0 & 1 & 1 & E_2 \\
1 & 0 & 1 & 0 & R_1 \\
0 & 1 & 0 & 1 & R_2
\end{bmatrix}.
$$

Clearly, the first four columns cannot be linearly dependent. Hence, degeneracy can only arise if the last column is linearly dependent on a subset of the others. Suppose, e.g., we have $\underline{a}$ x col 1 + $\underline{b}$ x col 2 = col 4 where $\underline{a}$ and $\underline{b}$ are any constants. Then, substituting, $\underline{a} + \underline{b} = E_1$, $\underline{a} = R_1$ and $\underline{b} = R_2$ so that we have $E_1 = R_1 + R_2$—a partial sum of requirements equals a partial sum of surpluses, as was asserted. This argument is easily extended into a formal proof.

To see how this effects the number of routes which must be used, consider the following two transportation problems the first of which is not degenerate while the latter is.

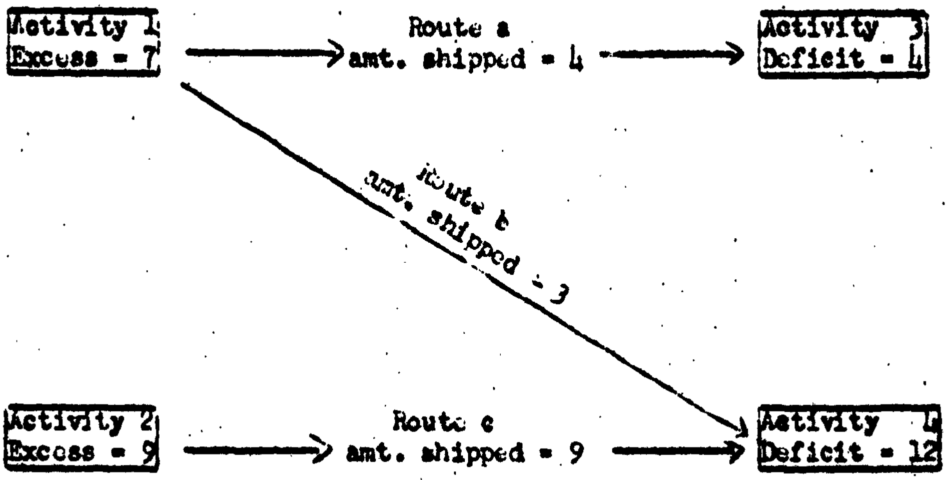
Problem 1

Activity	Excess	Requirement
1	7	
2	9	
3		4
4		12

Problem 2

Activity	Excess	Requirement
1	7	
2	9	
3		7
4		9

The first of these problems and one of its solutions may be represented schematically as follows:



Note that the (diagnol) shipping route, b, must be used because activity 1 cannot ship all of its surplus to activity 3. But in the degenerate problem the corresponding solution becomes:

Activity 1 Excess = 7	Route a amt. shipped = 7 →	Activity 3 Deficit = 7
Activity 2 Excess = 9	Route c amt. shipped = 9 →	Activity 4 Deficit = 9

Note that it has been possible to eliminate the diagnol route b which was used in the previous problem. This trivial example illustrates how degeneracy reduces the number of shipping routes which must be employed in solving the problem.

It is clear then, that while degeneracy has sometimes proved to be a nuisance in computation it can be extremely useful in a fixed charges problem. However, there are two reasons why looking for problems which happen to be degenerate is not a satisfactory expedient:

1. There is no guarantee that a significant proportion of the problems which are encountered in practice will turn out to be degenerate.1/

2. Even when a problem is degenerate, identification and consideration of all of its degeneracies is likely to be a long computational process. For, essentally, this requires the computation of all partial sums of excesses and of all partial sums of requirements and their comparison in order to see which if any of these partial sums happen to be equal. The number of combinations which is involved mounts very rapidly with the scale of the problem.

It was therefore decided to undertake a systematic extension of a current SPCC clerical procedure. Often clerks will simply decide that very small requirements or excesses are not worth the trouble of elimination by redistribution. In effect, this decision amounts to the elimination of a

---

1/In a sample of the 34 problems which on prior inspection appeared most likely to be degenerate, only five turned out to exhibit useful degeneracy. (Problems 9, 17, 19, 25 and 31 in appendix Table I.

fixed charge by forcing a degeneracy on to the problem. A small clerical change in requirements or excess figures has been used to eliminate one or more shipments.

Generalising this procedure, it will usually be possible to impose degeneracy on such problems by making insignificant changes in surplus and deficit figures which make some of their partial sums equal. This is clearly desirable, so long as the resulting savings in fixed charges are greater than any costs which are produced by changing the surplus or shortage figures. That is the approach which was taken by the algorithm which was developed in this study and which is described in detail below.

VII. The Approximation Methods Tested

Since detailed information on the actual magnitude of fixed costs has not as yet been made available to BUSANDA, it may not be possible for the moment to employ the fixed charge algorithm which has been developed. For the answers which it yields are, of course, dependent on the levels of these charges.

It therefore may be necessary, at least temporarily, to make use of a more routine transportation calculation until the required data are obtained. In any event, such a calculation constitutes an essential part of the fixed charges computation. It was necessary therefore to test out a variety of approximative computing methods for the ordinary transportation problem and to compare them with the results of a precise optimality calculation.

For this purpose, a sample of 100 actual SPCC redistribution transportation problems was collected and a number of tests were made with its help. The nature of the sample and the results of the calculations are described in detail in later sections of this report.

In addition to the optimal solutions and the proximity table solutions actually arrived at by SPCC, both as determined by the machine, and as adjusted by the clerks, the following two types of solution were investigated:

(a) The first approximation method which was tested was labelled ship most at least cost (SMALC). The basic idea is to find which route involves costs lower than any other's and to ship as much as possible along this route. Then we ship as much as possible along the second lowest cost route and so on until all excesses have been eliminated and all requirements have been filled.

Specifically, let $X_{i,j}$ represent the amount shipped from activity i to activity j, let $C_{i,j}$ be the unit cost of that shipment, let E_i represent the excess at activity i and R_j represent the requirement at activity j. Then the method proceeds as follows:

In the cost matrix choose the minimum cost figure $C_{i,j}$. Set $X_{i,j}$ = smaller of E_i and R_j and remove the corresponding row i or column j from consideration.

Replace $\quad E_i$ by $E_i - X_{i,j}$

$\qquad\qquad R_j$ by $R_j - X_{i,j}$

(Hence at least one goes to zero) and repeat on the new smaller matrix; continue until the problem is solved.

The basic idea is illustrated by the following tables. In the left hand table the entries on the outside represent activities excess quantities and requirement quantities. The spaces in the table represent shipping routes, and the numbers which are entered in these spaces represent the unit cost (distance) of a shipment along that route. For example, the 832 entered in the upper left hand corner represents the cost of shipping one unit from activity A to activity D.

		D	E	F	G
	Activity Requirement	18	301	52	4
A	267	832	771	940	2488
B	104	531	30	663	3247
C	4	3336	3380	3444	73

Activity Excess — Cost Table

	D	E	F	G
A	18	197	52	
B		104		
C				4

Solution Table

The minimum $C_{i,j}$ is clearly the 30 shipping cost from activity B (whose excess is 104 units) to activity E (whose requirement is 301 units). Hence, the maximum amount which can be shipped along this route is 104 units and this is the amount entered in the corresponding position in the right hand solution table.

Now activity B's excess figure is reduced to zero while E's requirement is reduced to 197. With this change in the original table we proceed to assign as large a shipment as possible (4 units) along the second lowest cost route (the 73 figure in the lower right hand corner), etc., until all requirements are met and all excesses are eliminated in the manner shown in the solution table.

(b) A second method of approximation which was employed is a modification of Vogels Approximation Method (VAM). This method is a bit more difficult to explain and involves more computer time. For each possible excess activity, i, one finds the lowest and the second lowest shipping cost routes which begin at activity i. Similarly one determines the lowest and second lowest cost routes which terminate at each requirement activity. The difference between these lowest and second lowest cost figures may be referred to as the error penalty which would result if the second lowest cost route were inadvertantly chosen. The basic idea of the VAM method is to seek to avoid these error penalties. Thus, now having an error penalty figure for each activity, we pick that activity which has the largest error penalty figure. On the argument that here is where the most expensive mistake can be made, we make as much of that activity's shipment as possible along its least cost route to avoid such a costly error. We now reexamine (and, if necessary, recompute) the error penalty figures and next take care of the activity with the largest remaining error penalty, and so on.

The row unit penalties = (61,501,3263)

The columns unit penalties = (301,741,307,2415)

The row absolute penalties = (16,287; 52,104; 13,052)

The column absolute penalties = (5,418; $\boxed{223,041}$; 15,964; 9,660)

Greatest Penalty

Hence the first shipment must be assigned in accord with second column minimum entry, i.e., the first shipment must go from activity A to activity B which, by coincidence, is the same as in the SMALC method.

More explicitly, the following procedure was employed.

Locate the minimum element in the cost matrix in each row and column. Call these

	Rows	Columns
(1)	$C_1 \ldots C_m$	$d_1 \ldots \ldots d_n$

Locate the next largest element in each row and column. Call these

(2)	$C'_1 \ldots\ldots C'_m$	$d'_1 \ldots\ldots\ldots d'_n$

Then the "unit penalties" incurred by not shipping on the routes located at the entries (1) are

$$C'_1 - C_1 \quad \ldots \quad C'_m - C_m, \quad d'_1 - d_1, \quad \ldots, \quad d'_n - d_n$$

Instead of these unit penalties, the computation employed "absolute penalties"

e.g. if $C_1 = C_{1,j}$ and is in row 1 and column j, we can only ship $X_{1,j} = \min(S_1, D_j)$ on this route.

Hence the "absolute penalty" is $X_{1,j} \; (C'_1 - C_1)$.

Shipments as large as possible are then made along routes where this absolute penalty is greatest.

<u>Example</u>:

Using the same problem as before, the cost table may be rewritten as

			D	E	F	G
			18	301	52	4
	A	267	832	771	940	2488
Cost matrix	B	104	531	30	633	3247
	C	4	3336	3380	3444	73

Where:

Row minima are underlined Thus ____

and

Column minima overlined Thus ____.

VIII. The Degeneracy Forcing Algorithm

The last method tested is the one which was especially designed for the fixed charges problem. It has these central features;

1. An adjusting device for excess and requirements figures which is designed to produce a reasonable amount of degeneracy.

2. An optimality computation device which is an extention of the SMALC method described in the previous section.

Specifically, the method is the following:

SMALC--Degeneracy Forcing Algorithm

Scan the matrix for $\min_{i,j} c_{ij}$. Suppose this is achieved at $i=k$ and $j=m$. If $|E_k - R_m| \leq \Delta$, set $X_{km} = \max$[1] $[E_k, R_m]$ and delete both row k and column m. If $|E_k - R_m| > \Delta$, as in the usual SMALC algorithm, set $X_{km} = \min [E_k, R_m]$ and then set

$$\left.\begin{aligned} E'_k &= E_k - X_{km} \\ R'_m &= 0 \end{aligned}\right\} \quad \text{if } \min [E_k, R_m] = R_m$$

and

$$\left.\begin{aligned} E'_k &= 0 \\ R'_m &= R_m - X_{km} \end{aligned}\right\} \quad \text{if } \min [E_k, R_m] = E_k.$$

1/Note that this always results in the "rounding up" of excess or requirement figures as needed. This decision was reached on the basis of SPCC opinions that cutting down of any such figures would only postpone them until the next period and would not eliminate the requirement or excess quantity in question. However the algorithm can easily be changed to work the other way by substituting "Min" for "Max" at this point.

Example (Using $\Delta = 1$)

Consider the following transportation cost matrix:

		4	4	6	2	4	2	Requirements
Excesses	5	9	12	9	6	9	10
	6	7	3	7	7	5	5
	2	6	5	9	11	3	11
	9	6	8	11	2	2	10

The preceding algorithm may readily be checked to yield the solution.

		4	4	7	2	4	3	Requirements
Excesses	5			5				7 shipments
	7		4				3	Amount shipped = 24
	2			2				Total cost = 126
	10	4			2	4		Average cost = 5.25

This may be compared with the following optimal solution obtained by the Simplex Method:

		4	4	6	2	4	2	Requirements
Excesses	5			5				9 shipments
	6		3	1			2	Amount shipped = 22
	2	1	1					Total cost = 112
	9	3			2	4		Average cost = 5.1

In this algorithm, Δ , the degeneracy limiting constant is the maximum amount by which excess or requirement figures are permitted to be revised in order to produce degeneracy. We recommend that a different Δ be employed for each item and that the value of each delta be revised at each review. The computation involved is very simple and the factors affecting the appropriate value of Δ can vary sharply over time and from item to item, so that a more inflexible Δ figure appears to be undesirable.

The value of Δ should be based on the values of the following two variables:

a. The average level of inventory on hand at the review date in those activities which carry the item. For, the higher the level of inventory on hand, the less significant, relatively, will be a given readjustment in requirement or excess figures. Hence Δ should vary directly with the average stock level.

b. The price of the item (as a rough index of military essentiality). Clearly, the more essential the item the less the adjustment in excess and requirement figures which can be permitted. Hence Δ should vary inversely with the price of the item.

In our trial calculation Δ was arrived at as follows. Define D by the expression

$$D = \frac{\text{Avg weekly system demand for the item}}{\text{Number of activities showing demand}} \times \text{a price adjustment factor}$$

Then Δ is equal to D after D has been rounded up to the nearest integer.

The average system demand was obtained from the current (May 15, 1960) CCSR page for the item. The number of activities showing any demand for this item in the past eight quarters was obtained from Code 740.

The price adjustment factor was developed on the argument that less essential items can be assigned larger Λ's, i.e., that it is appropriate to go further in forcing degeneracy on such items. The rule which was developed is summarized in the following table:

Price Adjustment Factor	Unit Price of Item [1]
1.5	\$100.01 and over
2.0	\$ 50.01 and \$100.00
2.5	Up to \$50.00

In effect, this means that for expensive items Λ is kept down to 1½ weeks of of average activity demand, similarly, for medium priced items Λ is set at 2 weeks demand etc.

Here is an example of the calculation:

F.S.N. HF 3010-318-9072 6.09 = Average Monthly Demand for System
Nomen. Clutch F 6 = Number of Activities Showing Demand
Unit Price \$24.50

DISTANCE TABLE					SOLUTION TABLE			
Consignors	Consignees			Excesses				
	75	85	91			75	85	91
71	893	3201	3203	1	71			
74	482	3014	3016	11	74	12		
83	3166	447	449	16	83		10	7
90	3380	664	666	1	90			
Requirements	12	10	7					

[1] The break points were developed partly on the basis of the information that more than 50 percent of SPCC shipments involve items worth less than \$50 while some 75 percent of the shipments involve items worth less than \$100.

The calculation of Λ :

$$D = \frac{6.09}{6X4} \times 2.5 = \frac{15.225}{24} \text{ So that } \Lambda = 1.$$

As a result, shipments from 71 and 90 were cancelled. The distance table was examined and the pair 83-85 was found to be the shortest haul. The amounts 16 and 10 showed a difference of 6, so the requirement at 85 was filled, leaving 6 units showing excess at 83. Now the pair 83-91 was looked at next. Here the difference between the 7 required and the 6 excess was 1 or equal to Λ. Thus the excess at 83 was considered to be 7 and the requirement at 91 filled. The final step was to find the difference between 74 and 75. The difference was again equal to Λ so 12 were assigned from 74, filling the requirement at 75.

The result of this use of Λ was to cancel out two shipments, satisfy all requirements with minor adjustments in shipping amounts from two activities.

We recognize that in some cases, Δ must be assigned the value zero. Such a case is:

F.S.N. HF 2910-217-0116
Nomen. Pump Fuel
Unit Price $352.00

Here cancelling out small shipments to requirement activities may clearly be highly undesirable. Only one pump may ever be required but it may be needed very badly. Some modification of Λ to take more explicit account of military essentiality is probably necessary. The degeneracy presently forced by clerical review at SPCC surely takes it into account.

Had it been possible to obtain the CSSR page that corresponded to the supply-demand review period in which these ordered shipments were made, substantially higher Λ's would probably have been used in the calculations. This is because we would have had a more accurate picture of the size of the excess stock at each shipping activity. Accordingly we could have found cases where increases in number of items shipped would have involved no danger to any such activity.

The lack of CSSR pages for these shipments was brought about by an unfortunate arrangement for the transfer of used CSSR pages from Stock Control, Code 740, to the Federal Documents Center at Mechanicsburg. The actions of one review period are, in practice, scattered through many different shipment orders, and trying to track them down is almost an impossible task. Because of this, the cost of obtaining this information would have been prohibitive. However, if the forced degeneracy algorithm is programmed into the SPCC review procedure, it will, of course, not involve any difficulty to take into account the data on the CSSR page. In that case information on inventory held at shipping activities can readily be used to change the magnitude of the price adjustment factor.

IX. Results of the Computations

The following table summarizes the results of the computations.

Table 1.--TRANSPORTATION "COSTS": RESULTS FOR AN AVERAGE TRIAL PROBLEM

	Computation method					
	SPCC ordered shipments	Simplex (optimum)	SMALC	VAM	Proximity table	Forced degeneracy
Cost (item-miles) 1/	1,127,000	1,115,000	1,119,000	1,132,000	1,123,000	1,119,000
Excess over optimum..............	12,600	0	4,100	17,100	8,700	4,700
Percentage excess over optimum (per problem)..........	2.60	0	0.45	1.60	1.95	0.20
Overall percentage excess over optimum..............	1.13	0	0.37	1.5	0.8	0.4
Standard deviation of percentage excess over optimum..............	5.50	0	1.30	3.30	4.50	4.60
Maximum absolute excess over optimum..............	215,000	0	111,000	628,000	164,000	88,000
Maximum percentage absolute excess over optimum......	29.20	0	7.20	21.30	28.90	19.70

1/ i.e. number of miles moved times the number of items in each shipment (added over all shipments).

These are all average figures for the 100 problem sample. For example, the first figure in the first column indicates that the average problem incurred 1,127,000 item-miles of transportation when redistributed in accord with the current SPCC decision process. This was an average of some 12,600 item-miles more than would have been involved in an optimal solution. For the average problem[1] this represents a 2.60 percent increase in transport costs over the optimal solution.

The last three entries in this column are meant to indicate the representativeness of these results and the largest deviations from them which have been encountered. As an index of the variability of the percentage increase in the cost figures we see that the standard deviation of that percentage figure is 5.50. Moreover, the largest absolute excess in cost for any of the sample problems of the SPCC calculation over the optimal solution is 215,000 item-miles. The largest percentage difference for any problem is 29 percent. These last two figures are meant to be indicative of the maximum risk incurred in using the approximation methods to solve a particular problem.

It is to be noted that (ignoring for the moment the Forced Degeneracy method) the SMALC method comes out best after the Simplex method on any one of the relevant criteria. It has the smallest excess in transportation cost over the Simplex result, taken either absolutely or percentagewise. Moreover the excesses have a smaller maximum variation and standard deviation than any other method. Roughly, we may conclude that the SMALC method will involve more than a 2 percent saving in transportation costs on the average redistribution as against present methods (this conclusion, of course, applies only to larger problems).

1/Notice this figure is not the percentage overall saving. Rather it is obtained by getting the percentage saving for each of the 100 problems and averaging them. This is clearly the arithmetic mean which must be used in computing the standard deviation.

It is to be noted that the Forced Degeneracy calculation, despite the fact that it involves about a 14 percent reduction in number of shipments for the average problem (Table 2), compares very favorably with SMALC in the variable transportation costs it involves.

Table 2.--TOTAL NUMBER OF SHIPMENTS (LINE ITEMS)

	In the Simplex solution	In the Forced Degeneracy solutions	Absolute difference	Percent difference	Average percentage difference
Total.	707	642	-65	-9.2	-14.3

This result is explained by the fact that the Forced Degeneracy method automatically eliminates some trivially small shipments and hence can result in an overall decrease in the total amount shipped in some problems. However, in general this method does involve considerable variability in the extent to which it approximates the transportation costs of the optimal solution. (In some problems its transportation costs will even be substantially below the Simplex cost figure.) Hence it seems advisable to maintain a conservative interpretation of the low average cost incurred by the Forced Degeneracy method. That is, it does not seem appropriate to consider this a reliable method for reducing transportation costs unless fixed costs are also substantial. It is remarkable that this method is able to achieve such satisfactory results with figures as moderate as those which were employed in the trial calculations.

X. Operations as SPCC

From the very beginning of our association with SPCC it was clear from the direct evidence as well as by reputation that the center was an efficient, well-run organization. Nothing we have had occasion to observe since that time has led us to revise this impression.

Naturally, any operation is certain to have some room for improvement, and this is certainly true of organizations whose rules and standards have evolved over time under the pressures of day-to-day operating needs. It is therefore possible and convenient for illustrative purposes to point out a few imperfections in SPCC operations. The purpose of such examples is to indicate the role which can be played by appropriate analytic procedures. They are not meant to convey a false impression of mismanagement in any part of the operation.

Indeed, our central results serve to point up the relative efficiency of SPCC operations. It is true that 75 percent of the large scale problems in our sample turn out to have been solved non-optimally. Nevertheless the solutions obtained by a combination of machine rule-of-thumb calculation and alterations based on clerical judgment, increased costs on the average by no more than three percent, which may well be considered a remarkable performance.

A. The Review Period

There is, however, one major feature of current SPCC inventory operations which is apparently under active reconsideration and which we believe merits careful examination. The period between reviews on redistribution and associated actions is approximately three weeks.

This is a considerably shorter period than that employed by other Navy supply operations, and it may well be too brief. Whatever savings in lower safety levels, and whatever speed in finding potential NIS situations may result from such a short review period must be weighed against possible undesirable effects. An excessively short review period can have several undesirable effects.

1. It adds to computing costs and to the pressure on computer facilities.

2. Unless specific provisions are made to avoid them, it can substantially increase the number of small shipments because excesses and requirements have not been given a chance to grow to an economic redistribution lot size. This can obviously lead to a rise in the outlay for fixed charges. There is evidence that a large number of small shipments has indeed been the result. In our sample, over 30 percent of the shipments involved lots of no more than five items and nearly half of them involved no more t an 10 units.

3. Excessively frequent reviews mean that fluctuations in demand may not be given a chance to offset one another. That is, if there is a temporary rise in sales at activity X and then a corresponding fall, at the end of this period the activity's inventory may well be in balance. However, too greouent a review might have led to goods being shipped to X at the end of the upswing in demand, in an effort to offset the apparently depleted inventory, and so activity X would end up in an excess position. Wherever such oscillatory demand behavior is fairly characteristic, therefore, there is a considerable advantage in infrequent review periods.

4. Excessively frequent reviews prevent the compounding of transportation problems in which economies can be achieved by optimal selection of routes. That is, if for one review period the stock status of one line item involves activities A B C and D and the next review period involves activities E F and G, it will normally be more economical to consider the excesses and requirements of all seven activities at once and to decide on shipping routes accordingly, rather than dealing with separate four-activity and three-activity problems. The latter is tantamount to taking a regular transportation problem and "simplifying" its solution by breaking it into two subproblems composed of arbitrarily chosen sets of activities.

Of course, this argument is not meant to imply that longer review periods are always preferable. There is clearly a limit beyond which longer review periods prevent the review from serving its purpose--the elimination of substantial inventory excesses and shortages. But we believe that a period longer than the present three week review is likely to be optimal, and is likely to result in considerable savings in transportation costs.

B. The Size Distribution of Problems at SPCC

One of the results of the short review period is that the redistribution transportation problems tend to be fairly small. This is illustrated by the following table representing a sample of 582 redistribution actions on or about May 19, 1959.

Number of activities involved either as consignor or consignee	Number of problems	Percent
2	296	50.8
3	91	15.6
4	85	14.6
5	36	6.2
6	36	6.2
7 or more	38	6.6
Total	582	100.0

This result is significant because, as already indicated, small problems offer relatively little scope for cost saving through systematic investigation of transportation routing. Indeed, any problem involving less than seven activities is rarely worth subjecting to a systematic transportation computation. For such a small problem the number of possible solutions is negligible and any reasonable method of arriving at a routing decision is very likely to yield an optimal or very near optimal solution.

Even where seven or more activities are involved they may not merit special computational procedures. If no more than two activities are involved as consignors (or as consignees) the problem is again likely to be computationally trivial. Thus in our sample, of the 38 problems involving seven or more activities, 12 turned out to be of this variety, so that only 26 or 4.5 percent of the problems resulting from one review period were of a size for which sophisticated computing methods are likely to be worthwhile.

C. Transportation Related Problems at SPCC

This suggests that at least with the current short review period, despite the gains that are shown in this report to be made possible by improved transportation routing calculation methods, these may not represent the most promising approach to increased efficiency in SPCC's redistributions. First, however, it should be remarked that this reservation does not apply to all of the Navy supply system. There are other portions of the system in which large transportation problems appear to be far more numerous and important and it is there that one would expect to find the most useful application of the results of the present study.

SPCC redistribution costs can doubtless be approached more effectively by extending the study to include an examination of activity requirement and excess figures, which were taken as given in the present investigation. These figures obviously play an essential role in the determination of redistribution costs.

For one thing, if inventory goals are taken literally and an activity is declared to be in a requirements or excess position whenever its stocks fall insignificantly below or above the target levels, an intolerably large number of shipments must result. The system is deprived of all flexibility and is made excessively responsive to changes in demand and inventory levels. Only by a systematic investigation of the costs and consequences of greater flexibility in excess and requirement criteria can an optimal arrangement be approached. Such an arrangement will be one which balances off the savings in transportation costs and associated fixed charges against the cost of decreased responsiveness in shipments to inventory level changes. In any event, it is clear that modifications in the excess and requirements criteria offer a promising prospect of achieving a reduced number of redistribution shipments.

Perhaps even more important, are the dynamic elements in the excess and requirements figures. Shipment decisions should ideally reflect not only current inventory levels, but also the expected magnitudes of these levels in the future. This is not a pure matter of the forecasting of demands. Current redistribution patterns themselves help to determine future inventory levels at the various activities. If redistribution decisions are based on current inventory levels a feedback mechanism is formed, in which inventory levels affect redistribution patterns, which in turn affect future inventory levels, and so on. A standard danger in such a situation is that unless specific preventative measures are taken, fluctuations in demand can lead to magnified fluctuations in inventory levels. The result is an artificially increased need for redistribution outlays, which is very likely to be substantial in magnitude. To avoid such wastes, excess and requirements figures must then be determined in a dynamic context in which their interrelations with future excesses and requirements are taken into account with the specific objective of reducing overall redistribution costs over time, and not just in the short run.

It is, then, our view that some of the most promising lines of investigation from the point of view of economy in SPCC's redistribution operations involve a systematic analysis of the redistribution process which emphasizes the excess and requirement determination procedures.

D. Current SPCC Redistribution Rules

Before concluding these remarks on current SPCC operations, it is convenient, for reference purposes to summarise the redistribution procedures which are currently being followed at SPCC in their current program of Supply-Demand Review. This program is being extensively re-written, and some of the current steps will be replaced. But these are the operations and the rules which are used at present.

Stock Status Review

Not much more than a year and a half ago the Stock Status Review was on a quarterly basis. Now, with the aid of the electronic computer[1] it has been changed to a tri-weekly schedule. Before describing the actions taken by the machine, it should be noted that the machine uses a slightly more complicated method to accomplish what was previously done by the clerks at SPCC. A number of the machine's calculations are checked out entirely or in part by the clerks in their review of the machine's recommendations. The machine computes recommendations and CSSR page changes more quickly than the clerks can consolidate the transactions and then make the calculations and the changes. But the clerks are now doing almost as much paper work as before, and perhaps even more.

As the daily transaction records are fed into the machine, each item for which there is an activity or system demand sets the machine in motion. The perpetual inventory tape and contract status record tape are updated. This occurs weekly. Once every three weeks the updated perpetual inventory and contract status tapes are run through with the calculating tapes. These results are the Supply-Demand review data.

For every item on which there is an activity or system requirement, i.e., for which a reorder point is reached, the machine goes through the procedures which will next be described. First it should be noted,

---

1/Since June 20, 1959, this is an IBM 705 Mark III.

however, that there may be a system requirement, and no activity requirement, when the normal system reorder point is reached or exceeded. The system reorder point is a function of the procurement lead time, and is automatically reached at preset intervals, depending upon stocking policy and lead times. But an activity's reorder point is reached when the number of months of supply of inventory which it has on hand falls below a fixed figure, known as its Requisitioning Objective Factor (ROF).

When a requirement sets the machine in motion it computes the system's and/or the activities' requirements or excesses using the following four concepts: Average monthly replenishable demand, requisitioning objective factor, variable safety level and stock status. These will now be defined.

A. Average monthly replenishable demand

I. For the System

1. Fast fraction (items with annual demand of greater than 11 units for entire system)

   a. 1.097 standard deviations normally
   b. 2.35 standard deviations for "Essentiality 1" items /predominantely load list items (considered to be hard core)/

   a normal curve is assumed to apply when the standard deviation is employed. The goals are 85 percent "Effectiveness" normally; 99 percent for load list and other E-1[1]/ items.

2. Slow fraction (less than 11 for system per year): 2.35 standard deviations are used in all cases. Here a Poisson distribution assumed to hold.

3. "X" fraction--not programmed.

4. All fraction system reorder point quantities are supposed to cover the lead time of system procurement.

1/Refers to Military Essentiality of a high order.

II. For an Activity

The machine allows two standard deviations on the basis of an assumed Poisson distribution constructed from the last 8 quarters' demand.

B. The Requisitioning Objective Factor for both system and activity

This factor is defined as the minimum quantity on hand and on order needed to sustain current operations and is composed of the operating and safety levels with procurement lead time or order and shipping time, as appropriate (the latter only for the system, not the activity--SPCC is investigating the desirability of also bringing it into the activity formula).

C. The Variable Safety Level

That is the quantity of inventory which will provide for minor interruptions in supply or changes in demand, for the system only. (Again SPCC is investigating ways of bringing a variable safety level into operation for the activities. Presently it is on a fixed months of supply safety stock level, with the system on a variable level.)

D. The Stock Status (System and Activity).

The Stock Status calculation examines

On-hand inventory
Due in (adjusted for delinquency of contract) by dates
Planned requirements established
Obligations established.

If any of the following five conditions for printing a CSSR page are present, the machine will produce an EDPM Action Form (Form 6210)1/. These five circumstances occur

1. When any activity has a requirement.
2. When the system has a requirement.
3. When any change occurs in the Requisitioning Objective Factor.
4. When the item is coded critical.
5. When the item is coded expedite.

1/At present the machine makes all these searches for all items in the inventory. But it prints out only the results of its search for "F" fraction items on EDPM Action Forms. These are treated as recommendations, and are reviewed by clerks. In addition, the clerks originate the needed redistributions for all "S", "R" and "X" fraction items during each Supply-Demand Review.

EDPM Action Forms are designed to reallocate or redistribute stock to correct projectable or actual shortages or excesses found to exist because of changes in demand, or to establish system procurement at the regular reorder point.

So, if there is an activity or system shortage, the machine

1. locates activities with shortages
2. locates activities with excesses
3. computes the system shortage or excess from these figures.

1. If there is a system shortage, the EDPM will print "critical" or "C" on the CSSR page which is the signal that something must be done to correct the situation immediately. Whenever an EDPM Action Form is marked "critical" the rule is that there will be no attempt made to redistribute. The item will be expedited from due-in contracts, if they exist, for the short activity, or from contracts due in at other activities, which if diverted would not subsequently leave them short. This is reallocation. Or a new contract would be set in motion with all possible speed. This is procurement allocation.

2. If there is no system shortage but there is an activity shortage, activities are recognized as being either short or in excess. The first step is to scan due-in contracts, as above, to find if the shortage can be overcome by expediting the due-in contract to the activity, or by reallocating from activities with excesses in their due-in contracts. This sorting through due-in contracts goes on until the requirement is satisfied or until all contracts with due dates before stocks at the activity will be exhausted are examined and all possible reallocations are made.

3. If there then remains an activity with a shortage, redistribution is considered.

Each excess stock position is examined again in light of the results of the reallocations to see if there is going to be enough excess to fulfill the requirement and not leave the activity with the excess in a short position before the next system buy. Requirements are filled from that excess which exists after the activities' issue rate, contract due-in date has been considered, necessary reservations made, and a "net" excess established. The decisions fall into two categories:

Case 3A. If the system excess is at least 24 M,[1/] starting in Area W[2/] with the activity with the greatest requirement, the machine seeks out possible consignors in order in Zone 1A looking for one with sufficient excess available to fill the total requirement. If no such consignor can be found in Zone 1A an attempt is made to satisfy the total requirement by partial redistribution in Zone 1A from consignors with excesses. In this case, shipments must be made in multiples of the intermediate pack[3/] size. If the requirement is not filled and the remainder is less than the amount which constitutes an intermediate pack, the balance is cancelled. If a requirement remains which exceeds the intermediate pack size, the machine tries the above steps for Zone 1B.

1/M is defined as the average monthly replenishable demand, calculated over the preceding 8 quarters. Thus 24 M equals 2 years' supply.

2/A possible explanation for this starting point is that the table was constructed in the East from where the West Coast seems very far off.

3/The intermediate pack size is the minimum number of packages of material which will be moved in an intraservice redistribution. Thus, if five pumps fit a packing case of convenient size, this is the minimum the Navy will send from one activity to another. The rule is "manufacturer's package (unit) times intermediate pack is the movement amount" (see exception in discussion of 3B below).

All activities with requirements in Area W will be satisfied as far as possible before cross-haul (east to west shipment) possibilities are investigated. Similarly, all possible requirements in Area E will be satisfied before cross-hauls are attempted. This is to preclude movement of material from the east coast to the west coast while leaving an unfilled requirement on the east coast.

Case 3B. If the system excess is 23 M or less, starting in Area W, the first activity with a requirement is, if possible, filled from activities in zones 1A and/or in 1B which have shown no current demands or no demands in the past 8 quarters. The intermediate pack size requirement is disregarded for this step. If this does not fill the requirement, the requirement is reworked into multiples of the intermediate pack (rounded off to the next higher unit). Then, the procedure tries to find an activity in Zone 1A with sufficient excess to fill the requirement. If not successful, it attempts a partial redistribution, cancelling the remainder under the unit times intermediate pack rule as in 3A. The remaining steps are similar to those in 3A.

In either case (3A and 3B) there will be another reallocation computation for all items with contracts outstanding or a procurement calculation will be undertaken to see if these redistributions have altered the reorder point.

The differences between these two ways of using the proximity table reflects this kind of thinking: When over supplies are generally great (3A) the greatest excesses should be eliminated first. But whenever supplies are generally small (3B) one should try to close out bins (i.e. remove stock from activities which have experienced zero demand for the last two years). When this does not work, then one should revert to the activity with greatest excess no matter the level of its current M.

It should be noted that only when activities with zero demands in 3B are involved, is the rule that moves should occur in intermediate pack size units violated. This is to allow all of the balance at the activity to be shipped out, and the bin to be closed.

4. The Proximity Table

From its name, it would appear that this table represents an alignment of consignors and consignees that are near to one another. Generall this is the case, as an examination of a map will show. But the present SPCC program is a translation into machine operation of what was previously done by clerical review. As a consequence, the proximity positions of certain pairs of activities are modified from what they should be in terms of either strict geographic distance or shipping cost. The proximity table thus clearly also takes other considerations into account. These may involve retraints upon the selection of activities which are in process of being liquidated, assigned special missions etc. In other words, the proximity table is a portion of a program put together from the logic of previous operations, rather than a piece of objective information.

This is brought out by comparing the current proximity table with an actual distance and a minimum shipping cost table as shown below:

CURRENT SPCC PROXIMITY TABLE
VERSUS MILEAGE AND LEAST-COST
TRANSPORTATION TABLES1/

	TO PUGET SOUND (71)	FROM					
Current Proximity Table		72	73	75	70	74	76
Mileage		72	75	73	70	74	76
Least-Cost transportation		73	75	72	74	76	70

1/Current SPCC Internal Instructions 4440.35B 1 Dec. 1958 "An Evaluation of Redistribution Decisions For General Stores Material", 1 Feb. 1956, Bayonne N.J. Tables S & T p.p. A-37; A-39. complete tables appear in appendix C as Tables III - 1,2,3.

In the first table, it is evident that Puget Sound (71) is closer in miles to NSC Oakland (75) than it is to NSY San Francisco (73). In SPCC's view it appears to be better to remove excesses from an industrial establishment (a navy shipyard) before they are eliminated from other activities. Moreover, it is desirable to remove stocks from activities with zero demand quickly, and 73 is more likely to satisfy this criterion than is 75. It is to be noticed how the clerical logic has been adopted by the machine.

Looking now at the last line of the first table it is clear that it is less expensive to ship to Puget Sound from San Francisco (73) and Oakland (75) than it is from Mare Island (72). The fact that these potential cost savings are not taken into account in the proximity table ordering of these activities must reflect SPCC experience, in the light of its desire to close bins, to give priority to shipyards in removing excesses. This is more striking when the position of Clearfield (70) is considered in this comparison. Both SPCC and the mileage table put Clearfield nearer Puget Sound than are San Diego (76) and Long Beach (74). But obviously, hauling over the mountains must be more costly than hauling material along the coast from a point as far distant as San Diego. In its present arrangement, SPCC must have been more impressed with distance than bin closing or perhaps larger shipment possibilities when it put Clearfield so high in its sequence over 74 and 76.

	TO MARE ISLAND (72)	FROM					
Current Proximity table		73	75	74	76	71	70
Mileage		75	73	74	76	70	71
Least-cost transportation		75	73	74	76	70	71

In the second table, in which Mare Island (72) is the consignee the proximity table places Puget Sound (71) before Clearfield (70), a clear difference between the mileage and cost ordering. Here removal of excess and bin closing must be the important reasons.

	TO PORTSMOUTH (81)	FROM								
Current proximity table		82	90	88	83	84	80	91	85	86
Mileage		82	90	88	83	84	80	85	91	86
Least-cost transportation		90	82	88	83	80	84	85	91	86

The comparison for Portsmouth (81) (the lower table) shows similar disparities among the three sequences. An examination of material relating to the East coast installations indicates that its ordering sequences resemble that of the West Coast.

Some interesting variations are found in the East-West hauls. A few examples are given below:

CURRENT SPCC PROXIMITY TABLE
VERSUS MILEAGE AND LEAST-COST
TRANSPORTATION TABLES[1]

	TO CHARLESTON (86)	FROM						
Current proximity table		70	74	76	75	73	72	71
Mileage		70	74	76	71	75	73	72
Least-cost transportation		70	72	73	75	71	74	76

	TO LONG BEACH (74)	FROM									
Current proximity table		86	80	91	85	84	83	88	90	82	81
Mileage		86	80	85	91	84	83	82	88	81	90
Least-Cost transportation		80	82	83	84	85	91	88	86	81	90

1/op. cit.

Noteworthy in this pair of comparisons is the fact that in the proximity table Charleston (86) must first have shipments assigned from Clearfield (70) and then from Long Beach (74) in their exact mileage sequence. Similarly Long Beach (74) first has shipments assigned from Charleston (86) and then from Mechanicsburg (80). Long Beach and Charleston are bases for special kinds of equipment (such as mine sweepers) and might be considered to have need for the same kinds of material. It is marked by the departure from the cross-haul pattern which holds for other activities under which East Coast installations first receive shipment from Clearfield and West Coast installations first receive from Mechanicsburg. The only other exception is San Diego which receives first from Charleston, and then from Mechanicsburg.

This discussion leads inevitably to the conclusion that the proximity table serves a variety of purposes only one of which is the objective of moving material between the two closest points. The other purposes of the table make it necessary to alter the position of a particular activity in a "proximity" sequence, thus increasing or decreasing its chances of being a consignor.

XI. Modified Linear Programming: Mathematical Supplement

The preceding sections have reported the course and conclusions of the present study. The purpose of this supplement is to supply certain additional mathematical considerations which are relevant to the investigation. Since the research involved a rather thorough survey of the techniques available for solving transportation problems, the results of this survey will be included. Due to the need for approximate solutions and the importance of the fixed charges in the present context (explained in Sections II and IV above), much of the literature is not directly applicable. However, in other situations different methods are certainly more appropriate and this compilation of methods may be of some use.

Within the field of linear programming, the transportation problem occupies a special position. It has the longest mathematical history (see, for example, [1] or [2] for transportation problems in disguise). It was formulated as a practical problem before the term "linear programming" was coined (see [3], [4], [5]). It covers, by ingenious interpretation, a greater range of problems than any other type of program (see, for example, [6], [7], [8], [9], [10], [11]). It has been solved in more ways than any other type of program. Our first task will be to classify these methods.

For this classification, we shall take the problem in its simplest mathematical form. This will avoid formal complications; however, it should be noted that most of the methods generalize easily to the various extensions of the problem that have been considered. Precisely, the problem to be solved has the form:

Minimize the linear form

(1) $$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{i,j} X_{i,j}$$

subject to the constraints

(2) $$X_{i,j} \geq 0 \qquad (i=1,\ldots,m;\ j=1,\ldots,n)$$

(3) $$\sum_{j} X_{i,j} = E_i \qquad (i=1,\ldots,m)$$

(4) $$\sum_{i} X_{i,j} = R_j \qquad (j=1,\ldots,n),$$

where the unit transportation costs $C_{i,j}$, the excesses E_i, and the requirements R_j are given data. We shall assume that $E_i \geq 0$, $R_j \geq 0$, and $\sum_i E_i = \sum_j R_j$, since these are necessary and sufficient conditions for the problem to have a solution.

1. EXACT ALGORITHMS

All seriously competitive algorithms make essential use of the following dual program and duality theorem (see [12] or [13]):

Maximize the linear form

(5) $$\sum_{i} E_i U_i + \sum_{j} R_j V_j$$

subject to the constraints

(6) $$U_i + V_j \leq C_{i,j} \qquad (i=1,\ldots,m;\ j=1,\ldots,n).$$

Theorem 1. The quantities $X_{i,j}$ solve the transportation problem if and only if they satisfy (2), (3), and (4) and there exist U_i and V_j satisfying (6) such that

(7) $$X_{i,j} > 0 \text{ implies } U_i + V_j = C_{i,j}.$$

In all of the exact methods proposed to date, at each stage of the computation, variables $X_{i,j}$, U_i, and V_j are present satisfying some subset of conditions (2), (3), (4), (6) and (7). The violated conditions are then used to alter the variables to improve one of the objective functions available (say (1) or (5)).

Primal Methods. In the methods of this class, one always works with quantities $X_{i,j}$ which satisfy (2), (3), and (4), that is, with feasible $X_{i,j}$. Then U_i and V_j are determined so as to satisfy (7) as well. The violation of condition (6) for some i and j calls for a change in the quantities $X_{i,j}$ so as to improve (1). This class includes the Simplex Method [14], the refinements of it due to Flood [15] and Gleyzal [16], and the version of the Simplex method popularly known as the Stepping Stone Method [17].

Dual Methods. Here one works with U_i and V_j which satisfy (7) in combination with the current choice of the primal variables $X_{i,j}$. The original algorithms of this class were devised for a special case of the transportation problem and are called the Hungarian Method (see [18] and [19]). The most practical versions for the transportation problem are due to Ford and Fulkerson (see [20], [21], [22], [23]); however, essentially the same ideas have been proposed by a number of authors ([24], [25], [26]).

Threshold and Feedback Methods. The method originated by Gerstenhaber [27], is based entirely on the dual variables U_i. These determine $X_{i,j}$ by means of intuitive "purchasing policies" for the activities with requirements which satisfy (2) and (4); violations of (3) then dictate the new choice of the U_i. As an approximation method this is known as the Threshold-Subsidy Method. A further modification, called the Feedback Method, was proposed in [28].

Analogue and Graphical Methods. In view of the fact that the exact machine programs are extremely fast, accurate, and accomodate very large programs, very little attention has been paid to analogue and graphical methods. A string-pulley arrangement is described in [29] and a pivot-bar device is explored in [30]. However, these can never be competitive in situations such as the application under consideration where speed is essential. A crucial feature to be noted is that these methods do not introduce new theory but merely mimic in a mechanical setting the methods used in the digital machine programs. The same conclusions apply to the graphical methods examined in [31] and [32].

Dynamic Programming. The application of dynamic programming to the transportation problem as proposed by Bellman in [33] has one fatal defect. It cannot handle problems with more than three activities in excess or with requirements without a prohibitively large amount of computer time and space.

In conclusion, our survey of the exact algorithms for the transportation problem leads us to the clear fact that there are only two currently competitive methods which have been tested thoroughly on many problems with readily available codes. The first of these is the Simplex Method (alias the Stepping Stone Method). This method is available in ready-made programs for almost all scientific computers; the SHARE Program name is NYTRI. The second method is the Ford-Fulkerson variation of the Hungarian Method and is available as SHARE code "IB TFL, the Transportation Problem—Flow Method," SHARE Memo PA 464. Particularly for large matrices, this programafferds substantial gains in time over NYTRI. However, for the present application, the problems are clearly too small and the computer time and space too limited to allow either to be used.

2. APPROXIMATE METHODS

Very little work has been done on approximate methods prior to this study (the work of Houthakker [34] is almost unique). This is not surprizing in view of the satisfactory state of the exact methods classified in the preceding section and the fact that very few organizations have a quantity of problems so large as to necessitate approximations. The natural place to start was the methods for constructing initial feasible solutions for the Simplex Method. These are described in detail above in Section V[illegible].

In this section, we shall record one relevant theoretical result which has, however, not been incorporated in the calculations. The reason that it has not been used is that, although it is fairly effective in reducing the transportation costs, it has the unfortunate effect of sometimes increasing the number of shipments. In common parlance, the result states: "Never use routes which intersect!"; it is based on the following explicit solution of a 2 by 2 transportation problem:

Theorem 2. Consider the 2 by 2 transportation problem with data:

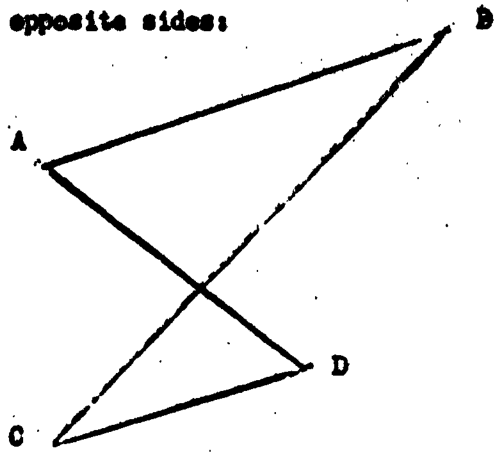
$$\begin{array}{cc} & \begin{array}{cc} R_1 & R_2 \end{array} \\ \begin{array}{c} E_1 \\ E_2 \end{array} & \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix} \end{array}$$

Suppose $C_{1,1} + C_{2,2} < C_{1,2} + C_{2,1}$ and that $E_1 \leqq R_1$. Then the following distribution is the unique solution:

$$\begin{pmatrix} X_{1,1} & X_{1,2} \\ X_{2,1} & X_{2,2} \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ R_1 - E_1 & R_2 \end{pmatrix}$$

Proof. This distribution is clearly feasible and $U_1 = -C_{2,1}$, $U_2 = -C_{1,1}$, $V_1 = C_{1,1} + C_{2,1}$, $V_2 = C_{1,1} + C_{2,2}$ satisfy Theorem 1. By assumption, $U_1 + V_2 = C_{1,1} + C_{2,2} - C_{2,1} < C_{1,2}$ and the solution is unique.

To derive our prohibition against "cross-hauling," one need only note that the sum of the diagonals of any quadrilateral is always greater than the sum of opposite sides:



$$\overline{AD} + \overline{CB} > \overline{AB} + \overline{CD}$$

Let the amounts shipped on this subgraph be

$$\begin{pmatrix} X_{AB} & X_{AD} \\ X_{CB} & X_{CD} \end{pmatrix}$$

and assume that a cross-haul has been made (i.e., that $X_{AD} > 0$ and $X_{CB} > 0$). We may assume that $E_1 = X_{AB} + X_{AD} \leqq X_{AB} + X_{CB} = R_1$ without loss of generality. Since the costs satisfy the assumption of Theorem 2, the unique minimum cost solution has $X_{AD} = 0$, which is a contradiction.

3. THE FIXED CHARGE PROBLEM

Any theoretical attack on the fixed charge problem must start from the two theoretical results of Hirsch and Dantzig [35]:

Theorem 3. If the underlying transportation problem is non-degenerate, and the fixed charges are positive and equal on all routes, then any basic optimal distribution (i.e., with m + n-1 routes in use) for the underlying transportation problem will solve the fixed charge problem.

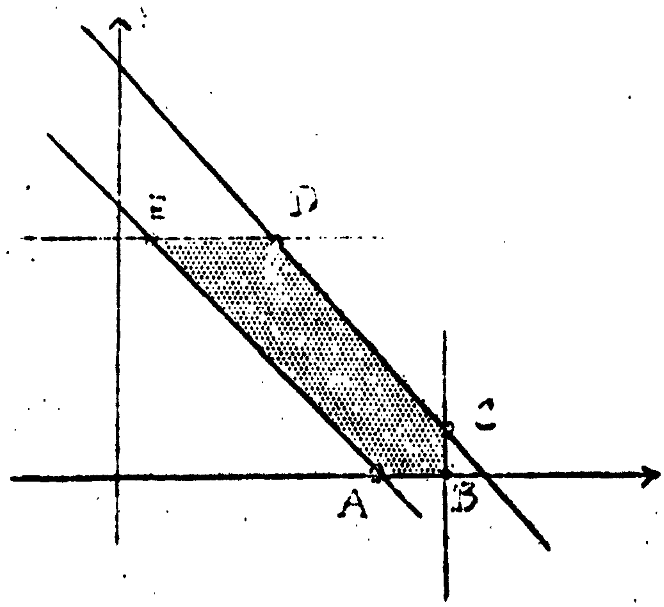
Theorem 4. In the general case, where the fixed charges may vary from route to route, an optimal distribution may always be achieved as a basic feasible distribution (although not necessarily optimal for the underlying transportation problem).

These results suggested an attempt to use an approach analogous to the Simplex Method, examing neighboring basic feasible distributions. The following example exhibits the difficulties inherent in such an approach:

$$\begin{array}{c} \\ E_1 \\ E_2 \end{array} \begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{pmatrix} C_{1,1} & C_{1,2} & C_{1,3} \\ C_{2,1} & C_{2,2} & C_{2,3} \end{pmatrix} \end{array} = \begin{array}{c} \\ 9 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccc} 8 & 5 & 3 \end{array} \\ \begin{pmatrix} 6 & 5 & 3 \\ 3 & 2 & 1 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} K_{1,1} & K_{1,2} & K_{1,3} \\ K_{2,1} & K_{2,2} & K_{2,3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 3 & 1 & 0 \end{pmatrix}$$

To exhibit the basic feasible solutions graphically, let $x = X_{1,1}$ and $y = X_{1,2}$. Then the feasible region is shaded below:



The extreme feasible distributions are tabulated below:

$$A = \begin{pmatrix} 6 & 0 & 3 \\ 2 & 5 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 8 & 0 & 1 \\ 0 & 5 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 8 & 1 & 0 \\ 0 & 4 & 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 4 & 5 & 0 \\ 4 & 0 & 3 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 5 & 3 \\ 7 & 0 & 0 \end{pmatrix}$$

The total costs are as follows:

A:	67
B:	66
C:	65
D:	67
E:	66

Thus E is a local minimum (the neighboring basic feasible distributions are A and D) but is not a global minimum.

It is inevitable that there can be no general theorem assuring an optimal solution covering the degenerate case, even when the fixed charges are constant, if the basic computational routine is approximate. However, somewhat trivially, we can be sure that we are moving in the right direction from a current approximation. This result will be dignified with the name of theorem although it is little more than common sense.

Theorem 5. Given a fixed charge problem with constant fixed charges on all routes. Let $X_{i,j}$ be a distribution with average transport cost A involving N shipments and let $X^*_{i,j}$ be a distribution with average transport cost A with $N^1 < N$ shipments. Then $X^*_{i,j}$ involves less total cost than $X_{i,j}$.

Proof: The total costs involved are

$$A \sum_i E_i + NK > A \sum_i E_i + N^1 K.$$

4. DEGENERACY

Recall that a degenerate distribution is one in which fewer than $m + n-1$ routes are used. Such distributions are possible if and only if there are two subsets A and B of the excesses and requirements, respectively, such that

$$\sum_{i \in A} E_i = \sum_{j \in B} R_j .$$

On the assumption that the average transportation cost resulting from the approximation techniques used is not changed significantly by slight alterations in the excesses and requirements, it is clear by Theorem 5 that forcing degeneracy decreases total costs. Two complementary remarks must be made in this supplement.

The first deals with the difficulties involved in <u>recognizing</u> rather than <u>forcing</u> degeneracy. A rough estimate of the number of comparisons needed to check the equality above is provided by $\frac{1}{2}(2^m - 2)(2^n - 2)$. (This is merely the number of non-trivial subsets of excesses compared with the number of non-trivial subsets of requirements. In one case, only one set of each complementary pair need be used; hence the factor of $\frac{1}{2}$.) This estimate can be reduced somewhat by a partial order of the subsets involved, building a subset one element at a time until it exceeds or equals a given comparison subset from the other class. Thus, we need never go past the point where the comparison subset is exceeded or equaled and a large number of comparisons are avoided. At best, however, the number of comparisons is prohibitive as only a <u>part</u> of a larger routine.

The second observation deals with the method adopted for forcing degeneracy. In any method of constructing a feasible solution which adds one shipment at each stage, in order to achieve the total of $m + n-1$ shipments, it must exhaust exactly one current excess or fulfill one current requirement until the last stage. Then, due to the balance equation $\Sigma E_i = \Sigma R_j$, both an excess and a requirement are cancelled. The method for forcing degeneracy proposed in the main body of this report is based on the fact that it cancels both an excess and a requirement with one shipment. The alterations in the given excesses and requirements are bounded by the factor Λ. This method is only the first in a class of methods in which higher order comparisons are made. E.g., we could ask:

$$\text{Is} \quad E_{i_1} + E_{i_2} - R_j \quad \leqq 2\Delta \ ?$$

If this holds, we would increase both E_{i_1} and E_{i_2} by an amount less than or equal to Λ and force a degeneracy. This is illustrated in the following example:

$$\begin{array}{cc} & \begin{array}{cc} R_1 & R_2 \end{array} \\ \begin{array}{c} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array} & \begin{pmatrix} X_{1,1} & X_{1,2} \\ X_{2,1} & X_{2,2} \\ X_{3,1} & X_{3,2} \\ X_{4,1} & X_{4,2} \end{pmatrix} \end{array} = \begin{array}{cc} & \begin{array}{cc} 8 & 7 \end{array} \\ \begin{array}{c} 3 \\ 3 \\ 3 \\ 6 \end{array} & \begin{pmatrix} 4 & 0 \\ 4 & 0 \\ 0 & 0 \\ 0 & 7 \end{pmatrix} \end{array}$$

$$\Delta = 1$$

Here E_1, E_2, and E_4 have been increased by Λ and E_3 deleted.

These higher order partial sums are not recommended for two reasons: (1) they require more complicated programming than seems feasible on the IBM 705 and (2) the order in which the partial sums are constructed is unlikely to coincide with the least cost entries which are at the heart of SMALC.

BIBLIOGRAPHY

1. D. König, "Über Graphen und ihre Anwendung auf Determinantentheorie und Mengenlehre," Math. Ann., 77 (1916), 453-465.

2. J. Egerváry, "Matrixok kombinatorius tulajdonságairól," Mat. Fiz. Lapok, (1931), 16-28, (translated as "Combinatorial Properties of Matrices" by H. W. Kuhn, ONR Logistics Project Princeton (1953), mimeographed.

3. F. L. Hitchcock, "Distribution of a product from several sources to numerous localities," J. Math. Phys. (M. I. T.), 20 (1941), 224-230.

4. L. Kantorovitch, "On the translocation of masses," Dokl. Akad. Nauk S. S. R., 37 (1942), 199-201. Reprinted in Management Science, 5 (1958-59), 1-4.

5. T. C. Koopmans, "Optimum utilization of the transportation system," in Proc. Int. Stat. Conf., 5 (1947), Washington, D. C. Reprinted in Econometrica 17 supplement (1949), 136-146.

6. A. S. Cahn, "The warehouse problem," Bull. Amer. Math. Soc. 54 (1948), 1073 (abstract).

7. G. B. Dantzig and D. R. Fulkerson, "Minimizing the number of tankers to meet a fixed schedule," Naval Res. Logist. Quart. 1 (1954), 217-222.

8. W. W. Jacobs, "The caterer problem," Naval Res. Logist. Quart. 1 (1954), 154-165.

9. W. Prager, "On the Caterer Problem," Management Science, 3 (1956), 15-23.

10. L. W. Smith, Jr., "Current status of the industrial use of linear programming," Management Science 2 (1956).

11. E. H. Bowman, "Production scheduling by the transportation method of linear programming," Operations Research, 4 (1956), 100-103.

12. D. Gale, H. W. Kuhn, and A. W. Tucker, "Chapter XIX of Activity Analysis of Production and Allocation," Cowles Commission Monograph No. 13, John Wiley and Sons, New York, 1951.

13. A. J. Goldman and A. W. Tucker, "Theory of linear programming," Annals of Math. Study No. 38, Princeton, 1956.

14. G. B. Dantzig, "Application of the simplex method to a transportation problem," Chapter XXIII of Activity Analysis of Production and Allocation, Cowles Commission Monograph No. 13, John Wiley, New York (1951), 359-373.

15. M. M. Flood, "On the Hitchcock distribution problem," Pacific J. Math., 3 (1953), 369-386.

16. A. Gleyzal, "An algorithm for solving the transportation problem," J. Res. N. B. S., 54 (1955), 213-216.

17. A. Charnes and W. W. Cooper, "The stepping stone method of explaining linear programming calculations in transportation problems," Management Science, 1 (1954-55), 49-69.

18. H. W. Kuhn, "The Hungarian Method for the Assignment Problem," Naval Research Logistics Quarterly, 2, Nos. 1 and 2 (1955), 83-97.

19. H. W. Kuhn, "Variants of the Hungarian Method for Assignment Problems," Naval Research Logistics Quarterly 3 (1956), 253-258.

20. L. R. Ford, Jr. and D. R. Fulkerson, "Solving the transportation problem," Management Science, 3 (1956), 24-32.

21. L. R. Ford, Jr. and D. R. Fulkerson, "Maximal flow through a network," Canadian J. Math., 8 (1956), 399-404.

22. L. R. Ford, Jr. and D. R. Fulkerson, "A simple algorithm for finding maximal network flows and an application to the Hitchcock problem," Canadian J. Math., 9 (1957), 210-218.

23. L. R. Ford, Jr. and D. R. Fulkerson, "A primal dual algorithm for the capacitated Hitchcock problem," Naval Research Logistics Quarterly, 4 (1957), 47-54.

24. J. Munkres, "Algorithms for the assignment and transportation problems," J. Soc. Ind. Appl. Math., 5 (1957), 32-38.

25. B. A. Galler and P. S. Dwyer, "Translating the method of reduced matrices to machines," Naval Research Logistics Quarterly, 4 (1957), 55-71.

26. W. Prager, "Numerical solution of the generalized transportation problem," Naval Research Logistics Quarterly, 4 (1957) 253-261.

27. M. Gerstenhaber, "A solution method for the transportation problem," J. Soc. Ind. Appl. Math., 6 (1958), 321-334.

28. H. W. Kuhn, "Methods for solving transportation problems," Techniques of Industrial Operations Research Seminar, Illinois Inst. of Tech., June 1957.

29. J. Stringer and K. B. Haley, "The application of linear programming to a large-scale transportation problem," Proc. First Int. Conf. on Operations Research, Oxford (1957), 109-122.

30. F. W. Sinden, "Mechanisms for linear programs," Operations Research, 7 (1959), 728-739.

31. M. L. Vidale, "A graphical solution to the transportation problem," Operations Research, 4 (1956), 193-203.

32. B. Zimmern, "Résolution des programmes linéaires de transport par la méthode de séparation en étoile," Revue de Recherche Opérationnelle, 1 (1957).

33. R. Bellman, "Notes on the theory of dynamic programming--transportation models," Management Science, 4 (1957-58), 191-195.

34. H. S. Houthakker, "On the numerical solution of the transportation problem," J. Operations Res. Soc. Amer., 3 (1955), 210-214.

35. W. M. Hirsch and G. B. Dantzig, "The fixed charge problem," RM-1383, December 1, 1954, RAND Corporation.

APPENDIX A

The Sample of Redistribution Problems

Originally SPCC instituted the tabulations from which Alderson Associates developed its sample of problems on which trial calculations were run. In the eight years, 1952 thru 1959, 85.59 percent of the value of SPCC replenishable demand occurred among 13,630 items. While this involves only 12.09 percent of the number of line items issued, it accounts for the bulk of the dollar cost of replenishable demand during this time.

These 13,630 items are the "best sellers" from among the 112,668 for which one or more demands were registered in eight years. We can assume that, being "best sellers", they are among the most active in the SPCC inventory.

From shipment orders (records of redistribution actions ordered by SPCC) covering April 1959 through September 1959, these "best selling" items were matched with all redistributions affecting them. A printout of many thousands of items resulted. From this printout, 100 test cases were selected in the following manner:

Two dates were selected at random from the review periods covered. They corresponded to the Supply-Demand Review of the 46th week of Fiscal Year 1959 (on or about May 19, 1959)[1] and the second week of Fiscal Year 1960 (on or about July 9, 1959).

---

[1] During the Supply-Demand Review of week 46 of FY 59, 6,870 redistributions were recommended by the EDPM review. Of these 2,132 were changed by clerical action, or 31 percent were either cancelled, partially cancelled or in other ways altered from the machine recommendations. For this review, 23,722 separate actions were recommended. Thus redistributions were about 29 percent of the work load.

From this sample, the first 100 redistributions encountered which were of sufficient magnitude were selected for the testing of our procedures. A distribution was considered of sufficient magnitude if it satisfied all three of the following criteria:

(a) it involved three or more consignor activities

(b) it involved three or more consignee activities

(c) it involved seven or more activities altogether.

APPENDIX B

Status of the Relevant Data

In its assignment Alderson Associates was not asked to develop any of the requisite cost data. These were to be obtained, insofar as they existed, from a variety of sources. It is appropriate therefore to comment briefly on the nature of the figures which became available to us. In summary it may be remarked that there is still a long way to go before the available information can be considered at all satisfactory.

A. Transportation Cost Data

By and large, most of the computations were based simply on distance tables rather than any direct transportation cost calculations. These figures have the advantage that they are readily available, they are straightforward and accurate.

Of course, the objective of a redistribution calculation is to save on transportation costs, not on mileage covered, and clearly these are not interchangeable data--freight rates have their own peculiar structure which is not readily explainable in terms of distance alone. It would therefore have been desirable to employ cost rather than distance tables in the analysis. However adequate transportation cost figures are difficult to come by for a variety of reasons.

1. Costs will vary by mode of transportation, and special rates apply to less than truckload and less than carload lots. Moreover, the mode of transportation employed is out of the hands of SPCC. Once a distribution action is ordered it is up to the base transportation officer to determine by what means it will be shipped, whether it will be combined with

other shipments, etc.[1] Thus SPCC never knows the cost of any proposed shipment in advance and no cost table can be made up which gives a firm figure for the cost of shipping a unit of item X from activity A to activity B.

2. Cost of transportation will vary from item to item. This means that, ideally, there should be a different cost table used for every item in the system. Clearly, this is out of the question. The data would be far too expensive and difficult to collect, and the computer simply could not cope with so much information in its review calculations.

3. It was therefore thought desirable to break items into groups with comparable transportation costs. OH has assembled information from which overall cost data were obtained by different federal stock groups. In particular, with the help of OH, experimental cost tables were constructed for the following 11 federal stock groups.[2]

FSG	Description
10	Weapons
20	Ship and Marine Equipment
28	Engines, Turbines and Components
29	Engine Accessories
43	Pumps and Compressors
48	Valves
53	Hardware and Abrasives
59	Electrical and Electrical Equipment and Components
61	Electric Wire, Power and Distribution Equipment
62	Lighting Fixtures and Lamps
66	Instruments and Laboratory Equipment

1/SPCC indicates the urgency of the request. Items needed badly get priority transportation.

2/Items in these categories made up 85 percent of SPCC ordered redistributions for period Nov. 1958 - Apr. 1959.

However it soon became clear that this was not an appropriate aggregation for present purposes. For example, Federal Stock Group 59 contains electrical and electronic equipment and components. There are 20 classes such as: 10, Capacitators; 40, Lugs, Terminals and Terminal Strips; 45, Relays, Contactors and Solenoids; 75, Electrical Hardware and Supplies, and; 99, Miscellaneous. Items within this group include:1/

Federal Stock Number	Nomenclature	Weight (in pounds)	Cube (cubic feet)	Price (in dollars)
HF 5910-695-4341	Capacitator	9.80	.146	38.50
HF 5940-284-4977	Terminal Box	9.50	.314	15.50
HF 5945-237-4678	Electrical Contact	.15	n.a.	.75
HF 5975-355-4748	Plug Stuffing Tube	.10	.005	.23
HF 5999-025-6379	Connector Switch	n.a.	n.a.	7.50
HF 5999-096-2715	" "	10.75	.305	47.00
HF 5999-197-4961	" "	5.00	.007	7.50

n.a. Not available.

This indicates clearly that transportation costs will vary considerably from item to item within such a group, so that it will not provide an appropriate basis for developing a small number of representative cost tables.

In sum, transportation cost tables which are satisfactory for redistribution computations have yet to be developed.

1/Information from CSSR page.

B. Fixed Charges

As is to be expected from their very nature, data on fixed charges are even more incomplete than those on transportation cost. In practice it is not always possible to identify a fixed charge from accounting records. Whether a charge is fixed or not depends, ultimately, on how it behaves when the scale of an operation is varied, and this therefore often cannot be deduced from observation of a given state of the system.

Our only information to date on the magnitude of fixed charges comesto us from three specific sources:

1. "A Proposal for Reducing Redistribution of General Stores Material," BUSANDA Project No. NT004010 25 Feb. 1955. Bayonne, N. J.
2. ALRAND reports SPCC.
3. Progress Report No. 2: "Preliminary findings NAS Quonset Point, R. I." 11 Dec. 1959, Dunlap and Associates, Inc. (and notes of report presentation by Dunlap in Washington, 29 Feb. 1960.)

Costs which are fixed per line item1/ regardless of size of shipment were estimated at $3.69. This figure came from the first source above. It consists of paperwork costs.

The procurement fixed costs were either $3.15 per line item if initiated by field activity or $4.18 per line item if initiated by GSSO, for local procurement; and were $18.08 per line item for central procurement.

1/A line item is the stock position of one item at one activity.

The ALRAND reports from SPCC give us the following values for fixed costs:

1. Procurement costs $25.00 per order (document, not line item).
2. A holding cost of 10 percent.
3. An obsolence rate of 20 percent.
4. A shortage cost estimated as the square root of the unit price, or derived from allowable risk, or on the basis of the Navy's accounting system for ships and operations, or the pure judgment of several specialists compared.
5. A figure sometimes called the "honking" charge, or the amount it costs SPCC to track down and eliminate a NIS or potential NIS situation, thought to be $12.00.
6. An estimated $9.00 per line item from Code 710 for a redistribution action for SPCC costs.

The Dunlap report isolated costs by departments of the Navy supply operation at the bases at which they did their work. They found that the majority of the cost totals were fixed over rather wide volumes of redistribution actions.

They also found wide differences between the cost of processing a supply action from one base to another. This cost difference was, in some cases, of the order of 6 to 1. These costs were in terms of dollars per line item, and the basic differences came about because of the different missions assigned the base in which the supply function was operating.

In the long run, the cost function was found to be a step-cost curve. Additional personnel are employed or reductions occur as work loads change in marked degree. In the short run there are no marked changes in number of persons employed. However, for such periods, Dunlap identified an additional fixed change factor, that of increased waiting time which results from the handling of additional documents or line items. It is not too clear from the available data whether or not an increase in the number of units distributed, while keeping the numbers of redistributions constant, would have an appreciable effect. Such an effect would doubtless be felt in some portions of the work of the base such as the breaking out of inventory, counting, packaging, etc. Waiting time would appear to be a cost which is fixed with respect to the number of items shipped for functions such as control, record-keeping, inventory balancing.

The Navy works on appropriations for fiscal years. Each operating unit gets a rather fixed amount of money to do its work for a year. As a result, the total cost of the Supply Department at NAS Quonset Point for fiscal year 1959 of 2.7 million dollars,[1] would limit the adjustments which could be made in personnel or equipments in the face of increased redistribution actions.

C. Costs of Procurement

Most incomplete of all is our information on procurement costs. Here the data problem is inherently so complex that there seems to be little immediate prospect of improvement. Procurement costs are highly dependent on the seller from whom an item is being obtained, his geographic location in relation to the activity to which delivery is to be made and the level of prices at the time the purchase is made. In addition, since a purchase adds to the system inventory, discounted carrying costs must be added to the purchase and transportation cost of the item, where the appropriate discount

---

1/P. 7. Dunlap and Associates Report No. 2 Op. Cit.

rate for the Navy supply system is by no means obvious (c.f. the discussion of the procurement cost coefficient in the model in section IV above). The available estimates on procurement costs come from the earlier Bayonne study of redistribution referred to in section B, above. SPCC uses an estimated $25.00 per document. Dunlap reports only on activity costs associated with the receipt of procurements, leaving the vast area of SDCP costs of procurement for a later study. At any rate it is clear that these costs are not available in sufficient detail and in a form which is suitable for a combined procurement, redistribution and reallocation computation.

APPENDIX C

Table I.—NUMBER OF SHIPMENT ROUTES REQUIRED BY DIFFERENT SOLUTIONS—SIMPLEX METHOD

Problem Number	Alternate Solutions Showing Number of Routes
1	8, 8
2	15
3	11, 11, 11
4	8
5	9, 9, 9
6	9
7	9, 9, 9, 9
8	8, 8, 8
9	8, 8, 9
10	12
11	9
12	6
13	8, 8
14	9, 9, 9
15	8, 8, 8
16	6
17	6, 7
18	6
19	8, 10, 9
20	8, 8, 8
21	6
22	11, 11, 11, 11, 11
23	9, 9, 9
24	6
25	7, 8
26	6
27	6
28	9, 9, 9
29	8
30	14, 14, 14, 14, 14
31	8, 7, 8, 8
32	9, 9, 9, 9, 9, 9, 9, 9, 9
33	6
34	7

Maximum total	288
Minimum total	282
Percent difference	2.1

Appendix C

Table II-A.--TRANSPORTATION "COST" FOR ALTERNATIVE SOLUTIONS

(In 000's of item-miles)

Problem number	SPCC ordered shipments	Simplex method	SMALC method	VAM method	Proximity table	Forced degeneracy method
1...........	64	60	60	60	61	59
2...........	279	235	247	285	246	250
3...........	4,157	4,095	4,157	4,151	4,112	4,157
4...........	5,408	5,352	5,365	5,363	5,408	5,365
5...........	2,912	2,870	2,870	2,923	2,905	2,870
6...........	204	160	161	178	171	162
7...........	1,615	1,610	1,610	1,616	1,629	1,617
8...........	417	386	387	389	417	390
9...........	7,000	6,987	6,999	7,030	7,065	6,999
10..........	10,859	10,644	10,646	10,727	10,808	10,646
11..........	156	135	136	142	159	136
12..........	104	98	99	103	99	99
13..........	59	58	58	58	58	58
14..........	129	127	128	127	128	128
15..........	801	757	763	761	801	763
16..........	204	201	201	227	201	203
17..........	30	29	29	30	30	28
18..........	73	72	72	73	78	72
19..........	61	58	59	59	59	57
20..........	114	110	111	113	111	107
21..........	280	231	231	257	280	164
22..........	242	241	241	242	242	241
23..........	40	39	41	40	46	40
24..........	43	39	40	41	40	40
25..........	250	249	251	264	250	251
26..........	247	242	245	242	247	258
27..........	4,916	4,912	4,913	5,541	4,916	4,913
28..........	849	848	848	849	849	854
29..........	853	816	816	821	848	816
30..........	489	378	379	379	488	371
31..........	489	482	483	483	482	488
32..........	851	795	795	795	795	795
33..........	94	78	79	79	84	83
34..........	252	249	249	250	252	249
35..........	83	82	83	85	83	85
36..........	136	135	136	136	136	136
37..........	3,340	3,197	3,286	3,197	3,200	3,286
38..........	1,428	1,420	1,420	1,429	1,428	1,425
39..........	17,961	17,841	17,841	17,950	17,857	17,841
40..........	431	429	429	435	431	429

Appendix C

Table II-A.--TRANSPORTATION "COST"
FOR ALTERNATIVE SOLUTIONS
(Continued)

(In 000's of item-miles)

Problem number	SPCC ordered shipments	Simplex method	SMALC method	VAM method	Proximity table	Forced degeneracy method
41.........	230	227	228	232	229	230
42.........	27	25	25	25	25	24
43.........	778	777	791	796	778	799
44.........	26	24	24	24	24	23
45.........	114	113	113	114	113	112
46.........	379	364	365	367	379	365
47.........	139	138	138	138	142	138
48.........	82	82	82	82	82	82
49.........	21	21	23	21	21	23
50.........	13	13	13	13	13	14
51.........	61	61	61	61	61	61
52.........	442	423	423	425	423	433
53.........	87	86	86	86	87	86
54.........	20	19	19	19	20	20
55.........	3,391	3,361	3,431	3,575	3,391	3,431
56.........	572	572	572	579	572	585
57.........	269	268	268	269	269	268
58.........	33	33	33	33	33	33
59.........	73	73	73	73	73	73
60.........	114	113	118	116	114	118
61.........	3,860	3,853	3,853	3,864	3,854	3,855
62.........	40	40	40	40	40	40
63.........	993	993	993	993	993	992
64.........	16	15	16	16	16	14
65.........	12	11	11	11	12	11
66.........	486	472	473	467	486	502
67.........	7	7	7	7	7	6
68.........	245	244	244	246	245	248
69.........	37	37	37	38	37	36
70.........	260	260	260	260	260	260
71.........	162	161	161	163	173	161
72.........	42	42	42	43	43	42
73.........	499	498	499	501	499	499
74.........	156	155	155	155	155	152
75.........	2,901	2,901	2,902	2,902	2,901	2,929
76.........	483	483	483	509	509	483
77.........	1,759	1,759	1,871	1,812	1,759	1,871
78.........	92	92	92	93	93	89
79.........	828	828	828	828	828	828
80.........	61	61	61	61	61	62

Appendix C

Table II-A.--TRANSPORTATION "COST" FOR ALTERNATIVE SOLUTIONS (Continued)

(In 000's of item-miles)

Problem number	SPCC ordered shipments	Simplex method	SMALC method	VAM method	Proximity table	Forced degeneracy method
81.........	10,796	10,796	10,796	10,901	10,796	10,796
82.........	393	393	393	416	393	393
83.........	70	70	70	75	70	70
84.........	242	242	242	242	242	242
85.........	23	23	23	23	23	23
86.........	1,356	1,356	1,356	1,358	1,358	1,356
87.........	86	86	90	86	86	90
88.........	932	932	932	932	932	932
89.........	7,902	7,902	7,902	7,916	7,902	7,902
90.........	24	24	24	24	24	29
91.........	65	65	65	65	65	64
92.........	219	219	219	219	227	219
93.........	117	117	117	117	117	117
94.........	232	232	232	232	232	232
95.........	14	14	14	14	14	13
96.........	80	80	80	80	80	80
97.........	27	27	27	27	27	27
98.........	1,519	1,519	1,519	1,533	1,525	1,519
99.........	837	837	837	843	837	837
100........	48	48	48	48	49	49
Total1/..	112,713	111,453	111,868	113,161	112,319	111,925

1/Slight differences may exist because of rounding.

Appendix C

Table II-B.--PERCENTAGE EXCESS OF TRANSPORTATION COST OF ALTERNATIVE SOLUTIONS OVER OPTIMUM (SIMPLEX) SOLUTION

(In percent)

Problem numbers	SPCC ordered shipments	SMALC method	VAM method	Proximity table	Forced degeneracy method
1..........	7.2	0	0	1.3	-0.6
2..........	18.3	4.8	21.3	4.7	6.2
3..........	1.5	1.5	1.4	0.4	1.5
4..........	1.1	0.3	0.2	1.0	0.3
5..........	1.5	0	1.8	1.2	0
6..........	26.8	0.2	11.1	6.4	1.2
7..........	0.3	0	0.3	1.1	0.4
8..........	7.7	*	0.6	7.7	0.9
9..........	0.2	0.2	0.6	1.1	0.2
10..........	2.0	*	0.8	1.5	*
11..........	15.5	1.0	5.0	18.3	1.0
12..........	5.5	0.3	5.1	0.3	0.3
13..........	1.3	0	0	0	0
14..........	1.6	0.5	0	0.5	0.5
15..........	5.9	0.9	0.5	5.9	0.9
16..........	1.7	0	13.0	0	1.1
17..........	3.0	0	1.6	3.0	-0.8
18..........	1.1	0	0.2	7.9	0
19..........	3.7	*	0.5	0.5	-1.8
20..........	2.8	*	2.0	0.5	-3.0
21..........	21.0	0	11.3	21.0	-28.9
22..........	0.1	0	0.5	0.1	0
23..........	0.3	3.6	0.3	15.4	0.5
24..........	9.2	0.3	2.3	0	0.3
25..........	0.3	0.7	6.1	0.3	0.7
26..........	2.1	1.2	0	2.1	6.4
27..........	0.1	*	12.8	0.1	*
28..........	0.1	0	0.1	0.1	0.7
29..........	4.5	0	0.5	3.9	0
30..........	29.2	0.2	0.1	28.9	-2.0
31..........	1.3	*	*	0	1.0
32..........	7.0	0	0	0	0
33..........	20.0	0.1	0.1	7.4	6.0
34..........	1.2	0	0.4	1.2	0
35..........	1.1	1.1	3.4	1.1	4.5
36..........	0.3	0	0	*	0.4
37..........	4.5	2.8	0	0.1	2.8
38..........	0.6	0	0.6	0.6	0.3
39..........	0.7	0	0.6	0.1	0
40..........	0.6	0	1.4	0.5	0

Appendix C

Table II-B.--PERCENTAGE EXCESS OF TRANSPORTATION COST OF ALTERNATIVE SOLUTIONS OVER OPTIMUM (SIMPLEX) SOLUTION (Continued)

(In percent)

Problem numbers	SPCC ordered shipments	SMALC method	VAM method	Proximity table	Forced degeneracy method
41..........	1.1	0.3	2.0	0.4	1.1
42..........	5.6	0	0	0	-4.2
43..........	0.1	1.8	2.4	*	2.8
44..........	11.9	0	0	0	-3.0
45..........	0.1	0	0.1	0	-0.4
46..........	3.9	*	0.7	3.8	*
47..........	*	0	0	2.6	0
48..........	0	0	0	0	0
49..........	0	7.2	0	0	7.2
50..........	0	0	0	0	0.4
51..........	0	0	0	0	0
52..........	4.6	0	0.6	0	2.4
53..........	0.1	0	0	0.1	0
54..........	6.5	0	0	6.5	0.6
55..........	0.9	2.1	6.4	0.9	2.1
56..........	0	0	1.2	0	2.1
57..........	0.3	0	0.3	0.3	0
58..........	0	0	0	0	0
59..........	0	0	0	0	0
60..........	0.3	3.4	2.2	0.3	3.4
61..........	0.2	0	0.3	*	*
62..........	0	0	0	0	0
63..........	0	0	0	0	-0.1
64..........	0.4	*	0.4	0.4	-2.2
65..........	10.2	0	0	10.2	0
66..........	3.1	0.3	3.2	3.1	6.5
67..........	0	0.3	0	0	-21.9
68..........	0.1	0	0.4	0.1	1.3
69..........	0	0	3.3	0	-0.1
70..........	0	0	0	0	0
71..........	0.5	0	0.8	7.5	0
72..........	0	0	1.7	1.7	0
73..........	*	*	0.4	*	*
74..........	*	0	0	0	-2.3
75..........	0	*	*	0	0.9
76..........	0	0	5.6	5.6	0
77..........	0	6.3	3.0	0	6.4
78..........	0	0	0.8	0.8	-3.8
79..........	0	0	0	0	0
80..........	0	0	0	0	0.5

Appendix C

Table II-B.--PERCENTAGE EXCESS OF TRANSPORTATION COST OF ALTERNATIVE SOLUTIONS OVER OPTIMUM (SIMPLEX) SOLUTION (Continued)

(In percent)

Problem numbers	SPCC ordered shipments	SMALC method	VAM method	Proximity table	Forced degeneracy method
81..........	0	0	1.0	0	0
82..........	0	0	5.7	0	0
83..........	0	0	6.9	0	0
84..........	0	0	0	0	0
85..........	0	0	0	0	0
86..........	0	0	0.1	0.1	0
87..........	0	3.9	0	0	3.9
88..........	0	0	0	0	0
89..........	0	0	0.2	0	0
90..........	0	0	0	0	19.7
91..........	0	0	0	0	-0.5
92..........	0	0	0	3.6	0
93..........	0	0	0	0	0
94..........	0	0	0	0	0
95..........	0	0	0	0	-4.5
96..........	0	0	0	0	0
97..........	0	0	0	0	0
98..........	0	0	0.9	0.4	0
99..........	0	0	0.7	0	0
100.........	0	0	0	0.1	0.8
Average1/.	2.60	0.45	1.60	1.95	0.20

1/Slight differences may exist because of rounding.

* Less than 0.1 percent.

APPENDIX-C.

TABLE III-1 SPCC PROXIMITY TABLE[1]

AREA W (WEST COAST)

Consignees	Zone 1 A	Zone 1 B	Zone 2 A	Zone 2 B
50	75 73 72 74 76	71 70	80 86 84 83	88 90 82 81 91 85
71	72 73 75	70 74 76	80 84 83	88 90 82 81 86 91 85
72	73 75 74 76	71 70	80 86 84 83	88 90 82 81 91 85
73	75 72 74 76	71 70	80 86 84 83	88 90 82 81 91 85
74	76 73 75 72	70 71	86 80 91 85 84 83	88 90 82 81
75	73 72 74 76	71 70	80 86 84 83	88 90 82 81 91 85
76	74 75 73 72	70 71	86 80 84 91 85 83	88 90 82 81
78	71 72 73 75	70 74 76	80 84 83	88 90 82 81 86 91 85

Consignees	AREA E (EAST COAST)		
81	82 90 88 83 84 80 91 85 86	70 75 73 72	74 76 71
82	81 90 88 83 84 80 91 85 86	70 75 73 72	74 76 71
83	84 88 90 80 82 81 91 85 86	70 75 73 72	74 76 71
84	83 80 88 90 82 81 91 85 86	70 75 73 72	74 76 71
85	91 84 83 80 86 88 90 82 81	70 74 76	75 73 72 71
86	91 85 84 83 80 88 90 82 81	70 74 76	75 73 72 71
88	90 82 83 81 84 80 91 85 86	70 75 73 72	74 76 71
90	88 82 81 83 84 80 91 85 86	70 75 73 72	74 76 71
91	85 84 83 80 86 88 90 82 81	70 74 76	75 73 72 71

1/SPCCINTINST 4440.35B 1 DEC. 1958.

TABLE III-2 MILEAGE BETWEEN NAVAL ACTIVITIES[1]

AREA W (WEST COAST)

Consignees	Zone 1	Zone 2
50	—	
71	78 72 75 73 70 74 76	80 84 83 92 85 91 82 88 86 81 90
72	75 73 74 76 70 71 78	80 84 83 92 82 88 85 91 81 90 86
73	75 72 74 76 70 71 78	80 84 83 92 91 85 82 88 86 81 90
74	76 75 73 72 70 71 78	86 80 85 91 84 83 92 82 88 81 90
75	73 72 74 76 70 71 78	80 84 83 92 85 91 82 88 86 81 90
76	74 75 73 72 70 71 78	86 80 85 91 84 83 92 82 88 81 90
78	71 72 75 73 70 74 76	80 84 83 92 85 91 82 86 88 81 90

Consignees	AREA E (EAST COAST)	
81	82 90 88 83 92 84 80 85 91 86	70 78 71 72 74 75 73 76
82	81 90 88 83 92 84 80 85 91 86	70 78 71 72 74 75 73 76
83	92 84 88 80 90 82 81 85 91 86	70 78 74 71 72 75 73 76
84	85 92 80 88 90 82 91 85 81 86	70 78 71 72 74 75 73 76
85	91 80 84 86 83 92 88 90 82 81	70 74 76 78 71 75 73 72
86	85 91 80 84 83 92 88 90 82 81	70 74 76 78 71 75 73 72
88	90 82 83 92 81 84 80 85 91 86	70 78 71 72 74 75 73 76
90	82 88 81 83 92 84 80 85 91 86	70 78 71 72 74 73 75 76
91	85 80 84 86 83 92 88 90 82 81	70 74 76 78 71 75 73 72
92	83 84 88 80 90 82 81 85 91 86	70 78 74 71 72 75 73 76

[1] Table "T", p. A39 An Evaluation of Redistribution Decisions for General Stores Material, Project NT00L010, Sub-project IR 54-3, 1 Feb. 1956. Source U. S. Government. Date 15 Dec. 1954.

TABLE III-3 MINIMUM FULL LOAD TRANSPORTATION RATES[1]

AREA W (WEST COAST)

Consignees	Zone 1	Zone 2
59	n.a.	
71	78 73 75 72 74 76 70	80 82 83 84 85 91 92 86 81 88 90
72	75 73 74 76 70 78 71	80 82 83 84 85 91 92 86 81 88 90
73	75 72 74 76 70 78 71	83 80 82 84 85 91 92 86 88 81 90
74	76 72 73 75 70 78 71	80 82 83 84 85 91 92 88 86 81 90
75	72 73 74 76 70 78 71	83 80 82 84 85 91 92 86 88 81 90
76	74 72 73 75 70 78 71	80 82 83 84 85 91 92 88 86 81 90
78	71 73 75 72 74 76 70	80 82 83 84 85 91 92 88 86 81 90

Consignees	AREA E (EAST COAST)	
81	90 82 88 83 92 80 84 85 91 86	70 71 73 74 75 76 78 72
82	88 83 92 90 81 80 84 85 91 86	71 72 73 74 75 78 76 70
83	88 82 90 84 92 85 91 80 81 86	70 71 72 73 74 75 78 76
84	85 91 83 92 80 88 90 82 81 86	70 71 72 73 74 75 78 76
85	91 84 80 83 92 86 82 88 90 81	70 71 72 73 74 75 78 76
86	85 91 80 83 92 88 82 90 81 84	70 72 73 75 71 74 78 76
88	83 92 82 90 81 84 80 85 91 86	70 73 74 75 71 76 78 72
90	81 82 83 92 88 84 80 85 91 86	70 71 73 74 75 76 78 72
91	85 84 80 83 92 86 82 88 90 81	70 71 72 73 74 75 78 76
92	88 83 82 90 84 85 91 80 81 86	70 71 72 73 74 75 78 76

1/Table "S", p. A37 An Evaluation of Redistribution Decisions for General Stores Material, Project NT00L010, Sub-project LR 54-3, 1 Feb. 1956. Minimum figures used to order above sequence were either rail or truck, whichever cheaper, prepared at Bayonne, N. J. by Navy Central Freight Control Office 1 Jan. 1955.

n.a. Not available.

IV. Representative Problems

These four problems represent an example of each of the following:

Problem a. Outstanding improvement in reducing transportation.

Problem b. Small but important improvements in transportation.

Problem c. High degree of degeneracy.

Problem d. A large number of alternate solutions.

There are seven tables shown for each problem. They represent; 1. the transportation distance between activities with the excess and requirement at each activity; 2. the SPCC ordered shipments; 3. the solution using the Simplex method; 4. the solution using the SMALC method; 5. the solution using the VAM method; 6. the solution using the present SPCC proximity rules (solution 2 contains variations from clerical review and this solution ignores these changes); and 7. the solution using forced degeneracy.

Problem a.

F.S.N. HF 2815-364-4345
Nomen. Cover He
Unit Price $15.10

Average monthly system demand 7.13
Activities showing demand 10
A 1

1. Transportation Table

Excess	Consignor	Consignee 50	76	86	90
10	70	3163	940	1991	2506
7	71	3285	1440	3262	3320
18	74	2874	102	2948	3336
51	75	2394	591	3299	3380
18	82	5669	3371	979	73
20	83	5558	3230	747	447
1	85	5623	3122	396	664
	Requirement	48	28	16	33

2. SPCC Ordered Shipments

	50	76	86	90
70	10			
71	7			
74	18			
75	13	28		10
82				18
83			16	4
85				1

(Result: 203,519)

The result of the SPCC ordered shipments is expressed as the sum of the number of units X the distance each moved. Each subsequent method shows a result calculated in the same manner.

3. Simplex Solution

Consignee

Consignor	50	76	86	90
70			10	
71		7		
74		18		
75	48	3		
82				18
83			5	15
85			1	

(Result: 160,565)

4. SMALC Solution

	50	76	86	90
70			10	
71	7			
74		18		
75	41	10		
82				18
83			5	15
85			1	

(Result: 160,873)

5. VAM Solution

Consignee

Consignor	50	76	86	90
70			10	
71	7			
74	18			
75	23	28		
82				18
83			5	15
85			1	

(Result: 178,351)

6. Proximity Table Solution

	50	76	86	90
70		7		3
71				7
74		18		
75	48	3		
82				18
83			15	5
85			1	

(Result: 170,913)

7. <u>Forced Degeneracy Solution</u>

Consignor	50	76	86	90
70			11	
71	7			
74		18		
75	41	10		
82				18
83			5	15
85				

(Consignee columns 50, 76, 86, 90)

(Result: 162,468)

Problem b.

F.S.N. HF 2910-261-2778
Nomen. Nozzle F
Unit price $5.10

Average monthly system demand 214
Activities showing demand 6
Δ 83

1. Transportation Table

Excess	Consignor	Consignee 74	75	83
50	71	1338	893	3136
18	73	495	8	3171
20	80	2949	2963	187
60	86	2948	3299	747
10	90	3336	3380	214
Requirement		68	60	30

2. SPCC Ordered Shipments

	74	75	83
71	50		
73	18		
80			20
86		60	
90			10

(Result: 279,630)

3. Simplex Solution

Consignor	Consignee 74	75	83
71	8	42	
73		18	
80			20
86	60		
90			10

(Result: 231,114)

4. SMALC Solution

	74	75	83
71	8	42	
73		18	
80			20
86	60		
90			10

(Result: 231,114)

5. VAM Solution

	74	75	83
71		50	
73	18		
80	20		
86	30		30
90		10	

(Result: 257,190)

6. Proximity Table Solution

	74	75	83
71	50		
73	18		
80			20
86		60	
90			10

(Result: 279,630)

7. <u>Forced Degeneracy Solution</u>

	74	75	83
71		60	
73			
80			
86	30		30
90			

(Result: 164,430)

Problem c.

F.S.N. HF 2910-336-9905
Nomen. Hydrault
Unit price $141.65

Average monthly system demand 6.75
Activities showing demand 8
Δ 1

1. Transportation Table

Consignee

Excess	Consignor	75	85	86	91
2	71	893	3201	3262	3203
15	72	30	3271	3339	3273
1	73	8	3236	3304	3238
2	74	482	3014	2948	3016
4	76	591	3122	3056	3124
1	83	3166	447	747	449
1	90	3380	664	964	666
	Requirement	6	12	2	6

2. SPCC Ordered Shipments

	75	85	86	91
71	2			
72	3	10	2	
73	1			
74				2
76				4
83		1		
90		1		

(Result: 60,911)

3. Simplex Solution (a, b, c)

	75	85	86	91
71		2		
72	6	9		
73		1		
74			2	
76				4
83				1
90				1

(Result: 58,764)

Simplex (d)

	75	85	86	91
71		2		
72	6	7		
73		1		
74		2		
76			2	2
83				1
90				1

(Result: 58,764)

3. (cont) Simplex (e,f,g,h)

Consignor	75	85	86	91
	Consignee			
71		2		
72	6	7		2
73		1		
74		2		
76			2	2
83				1
90				1

(Result: 58,764)

4. SMALC Solution

	75	85	86	91
71		2		
72	5	4		6
73	1			
74			2	
76		4		
83		1		
90		1		

(Result: 58,777)

5. VAM Solution

Consignor	75	85	86	91
	Consignee			
71		2		
72	6	5	2	2
73		1		
74		2		
76				4
83		1		
90		1		

(Result: 59,032)

6. Proximity Table Solution

	75	85	86	91
71				2
72	5	4	2	4
73	1			
74		2		
76		4		
83		1		
90		1		

(Result: 59,045)

7. **Forced Degeneracy Solution**

Consignee

Consignor	75	85	86	91
71		2		
72	6	10		
73				
74			2	
76				4
83				
90				

(Result: 57,684)

Problem d.

F.S.N. HF 2815-125-8069
Nomen. Joint AS
Unit price $13.00

Average monthly system demand 42.48
Activities showing demand 14
Δ 2

1. Transportation Table

Consignee

Excess	Consignor	85	88	91
102	74	3014	3281	3016
88	75	3231	3295	3233
70	81	736	169	738
41	82	679	112	681
25	84	355	223	357
325	86	396	879	398
53	90	664	97	666
	Requirement	15	88	601

2. SPCC Ordered Shipments

	85	88	91
74	15		87
75		88	
81			70
82			41
84			25
86			325
90			53

(Result: 850,716)

3. Simplex Solution (a)

	85	88	91
74	15		87
75			88
81		70	
82		18	23
84			25
86			325
90			53

(Result: 795,188)

Simplex (b)

	85	88	91
74			102
75	15		73
81		70	
82		18	23
84			25
86			325
90			53

(Result: 795,188)

Simplex (c)

	85	88	91
74			102
75			88
81	15	55	
82		33	8
84			25
86			325
90			53

(Result: 795,188)

3. (cont.)

Simplex (d)

Consignor \ Consignee	85	88	91
74			102
75			88
81	15	47	8
82		41	
84			25
86			325
90			53

(Result: 795,188)

Simplex (e)

	85	88	91
74			102
75			88
81		62	8
82	15	26	
84			25
86			325
90			53

(Result: 795,188)

Simplex (f)

	85	88	91
74			102
75			88
81		47	23
82		41	
84	15		10
86			325
90			53

(Result: 795,188)

Simplex (g)

	85	88	91
74			102
75			88
81		47	23
82		41	
84			25
86	15		310
90			53

(Result: 795,188)

Simplex (h)

	85	88	91
74			102
75			88
81		47	23
82		41	
84			25
86			325
90	15		38

(Result: 795,188)

Simplex (i)

	85	88	91
74			102
75			88
81		9	61
82		41	
84			25
86			325
90	15	38	

(Result: 795,188)

4. SMALC Solution

Consignor	Consignee 85	88	91
74			102
75			88
81			70
82		35	6
84	15		10
86			325
90		53	

(Result: 795,188)

5. VAM Solution

	85	88	91
74			102
75	15		73
81		70	
82			41
84			25
86			325
90		18	35

(Result: 795,188)

6. Proximity Table Solution

	85	88	91
74			102
75			88
81			70
82		35	6
84	15		10
86			325
90		53	

(Result: 795,188)

7. Forced Degeneracy Solution

	85	88	91
74			102
75			88
81			70
82		35	6
84	15		10
86			325
90		53	

(Result: 795,188)